UNIVERSITY
OF NAIROBI

SPH 101

MECHANICS -I

LECTURE SERIES

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Lecture notes adapted from Principles of Mechanics I
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INTRODUCTION

Why Study Physics?

For two reasons. First, Physics is one of the most fundamental of the sciences. Scientists of all disciplines make use of the ideas of physics, from chemists who study the structure of molecules to paleontologists who try to reconstruct how dinosaurs walked. Physics is also the foundation of all engineering and technology. To design a spacecraft or a better mousetrap, one has to understand the basic laws of physics.

But there is another reason. The study of physics is an adventure. You will find it challenging, sometimes frustrating, occasionally painful, and often richly rewarding and satisfying. It will appeal to your sense of beauty as well as to your rational intelligence.

Our present understanding of the physical world has been built on the foundations laid by scientific giants such as Galileo (1564-1642), Isaac Newton (1642-1727), James Maxwell (1831-1879), Michael Faraday (1791-1867) and Albert Einstein (1879-1955), and their influence has extended far beyond science to affect profoundly the ways in which we live and think. If you've ever wondered why the sky is blue, how radio waves can travel through empty space, or how a satellite stays in orbit, you can find the answers by using fundamental physics. Above all, you will come to see physics as a towering achievement of the human intellect in its quest to understand our world and ourselves. Indeed, Newton himself said, "*If I have been able to see a little further than other men, it is because I have stood on the shoulders of giants*".

The Nature of Physics

Physics is an Experimental Science. Physicists observe the phenomena of nature and try to find patterns and principles that relate these phenomena. These patterns are termed Physical Theories or, when they are very well established and of broad use, Physical laws or Principles.

Physics is not simply a collection of facts and principles; it is also the *process* by which we arrive at general principles that describe how the physical universe behaves. The development of physical theory requires creativity at every stage. *The physicist has to learn to ask appropriate questions, design experiments to try to answer the questions, and draw appropriate conclusions from the results.*

An essential part of the interplay of theory and experiment is learning how to apply physics principles to a variety of practical problems. Learning to solve problems is absolutely essential; *You don't know Physics unless you can do Physics*. This means not only learning the general principles, but also learning how to apply them in specific situations.

Objectives of the Course

This unit course is designed for a one semester (35 one-lecture hours) divided into six (6) Modules. It is intended to:

- provide a knowledge of essential principles of mechanics and their applications, relevant to a science student whether or not the student will study physics beyond the 1^{st} year of his/her studies.

- recognise the contribution of physics principles and appreciate their applicability and limitations in other disciplines and in everyday life.

- develop the student's creativity through qualitative and quantitative approach to the ideas of physics.

- develop attitudes relevant to physics such as;
  (i) concern for accuracy, precision, objectivity and integrity;
  (ii) the skills of enquiry, initiative and inventiveness;

- stimulate students and create a sustained interest in physics so that the study of the subject is enjoyable as a satisfying intellectual discipline

- familiarize the student with the use of computing facilities as an analytical tool to problem solving via simulation and designed experiments (where applicable)

Assessment Objectives

The assessment objectives listed below reflect those parts of the aims that will be assessed in the examination

A: Knowledge with understanding

- Students should be able to demonstrate knowledge and understanding in relation to

(i) Scientific phenomena, facts, laws, definitions, concept, theories;
(ii) Scientific vocabulary, terminology, conventions (including symbols, quantities and units)
(iii) Scientific instruments and apparatus, including techniques of operation, ability to design and evaluate an experiment to investigate a problem
(iv) Scientific and technological applications with their social, economic and environmental implications e.g., appreciation of the relevance of physics to everyday situations such as in the home.

Questions testing these objectives will often begin with one of the following words: *Define, state, describe, explain* or *outline.*

B: Competence in handling, applying and evaluating information

- Students should be able to

(i) Locate, select, organize and present information from a variety of sources
(ii) Translate information from one form to another

(iii) Use information to identify patterns, report trends and draw inferences and report conclusions;
(iv) Present reasonable explanation for phenomena, patterns and relationships
(v) Make predictions and put forward hypothesis
(vi) Apply knowledge, including principles, to novel situations
(vii) Evaluate information and hypothesis

Questions testing these objectives will often begin with one of the following words: *predict, Suggest, deduce, calculate* or *determine*.

C: Experimental skills and investigations

- Students should be able to

(i) Follow a detailed set or sequence of instructions
(ii) Use techniques, apparatus and materials, safely and effectively
(iii) Make and record observations, measurements and data with due regard for precision, accuracy and units;
(iv) Interpret and evaluate observations and experimental data;
(v) Evaluate methods and techniques and suggest possible improvements

Scheme of Assessment

- **Theory Paper**: (60 - 70% of the total marks). Candidates will be required to enter for an Examination consisting of a variable number of structured questions (e.g. five) of which they may have a choice of attempting three questions out of five.

- **Laboratory experiments**: (15 - 20%). Students should perform at least one laboratory experiment per week. Where possible, they should also be assessed on numerical problem solving techniques via computer simulations.

- **Continuos Assessment Tests (CATS)**: (15 - 20%). This should comprise of at least two tests and a number of homework and/or tutorials.

Module One: KINEMATICS

Space Time and Matter
Physical Quantities & Units
Scalars and Vectors
Rectilinear Motion
Projectile Motion
Circular Motion

USEFUL RELATIONSHIPS

Rectilinear/Translational Motion

Equations of motion; $v = u + at$

$$s = ut + \tfrac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Circular motion

Linear (Tangential) Velocity; $v = \frac{ds}{dt} = r\omega$

Angular Velocity; $\omega = \frac{d\theta}{dt} = \frac{v}{r}$

Angular acceleration; $\alpha = \frac{d\omega}{dt} = \frac{1}{r}\frac{dv}{dt}$

Tangential acceleration; $a_{\tan g} = r\alpha = \frac{dv}{dt}$

Centripetal Acceleration; $a_{cent} = \frac{v^2}{r} = \omega^2 r = \omega v$

Centripetal Force; $F_{cent} = \frac{mv^2}{r} = m\omega^2 r$

1.0 KINEMATICS

1.0 Introduction

What is mechanics?
The word mechanics comes from the Greek word *mechane* meaning machines. Since the essential aspect of the study of machines is motion, ***mechanics is thus a branch of physics that deals with the study of motion or change in position of a physical object***. It can be subdivided into THREE main categories, namely:

(i) ***Kinematics***: Concerned with the mathematical description (geometry) of motion of an object in terms of *position, velocity and acceleration*.
(ii) ***Dynamics***: Concerned with the physical causes of motion of a physical object.
(iii) ***Statics***: Concerned with conditions under which no motion is apparent i.e., conditions under which a physical object is in equilibrium.

This chapter begins with introducing the concepts of space, time and matter since these are very important aspects in physics

1.0.1 Space, Time and Matter

The concept of space and time is fundamental to the study of mechanics. All objects occupy space and have length, breadth and height. Space is thus 3-dimensional and measurements in space involve the concepts of length such that the position of an object can be specified completely by the three (3) co-ordinates (x, y, z) respectively.

1.0.2 Space

The following are the classical (Newtonian) properties of space:

- Space is flat (Eucledian flatness): In such a space, the shortest distance between two points is a straight line. However, from recent theories, space is not exactly flat but somewhat curved. The departure from flatness is however small and can be ignored in classical mechanics.
- Space is homogeneous: Space is everywhere alike and as such, the results of an experiment are not altered due to linear displacement of the co-ordinate system.
- *Space is isotropic*: There is no preferred direction in space. One direction is as good as the other.

1.0.3 Time

This concept (measured by clocks) is derived from our experience of having one *event* taking place after, before or simultaneously with another *event*. Absolute time and mathematical time by itself and from its own nature, flows equally without relation to anything external and is otherwise called duration.

The following are the Newtonian properties of time

- *Time is 1-dimensional*: Time flows only in one direction and can be specified by a single variable *t*. It is independent of space.
- *Time is homogeneous*: Time flows uniformly e.g. the results of an experiment is independent of the change in the origin of time i.e., the results of an experiment carried out 10 years ago should be the same as those obtained today under the same conditions.
- *Time is isotropic*: Time has no preferred direction and the laws of physics remain unaltered if $+t$ is changed to $-t$.

1.0.4 Matter

Physical objects are composed of "*small bits of matter (that which occupies space)*" such as atoms or molecules. The concept of a *particle is a material object that can be considered as occupying a point in space and perhaps moving as time goes by*. A measure of the *quantity of matter* associated with a particle is its *mass*.

1.1 Physical Quantities

Assessment Objectives

- *At the end of this section, candidates should be able to*

(i) *Recall that all physical quantities consist of a numerical magnitude and a unit*
(ii) *Recall the following base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol)*
(iii) *Express derived units as products or quotients of the base units*
(iv) *Use base units to check homogeneity of physical equations*

1.1.1 What is a physical Quantity

- It is a measurable entity of an object and consists of magnitude (e.g., how big) and a unit of measurement (representing the entity or property measured). For example, the length of an object may be expressed as 5 cm where 5 is the magnitude and cm is the unit of measurement.
- The most commonly used system of units is the international adopted system of units (*Système International* or *S.I.*). S.I units are founded upon seven internationally agreed *base units* (see Table 1-1).
- Base units are the units of a base quantity and no one base unit depends upon any other.

1.1.2 Dimensions

- A dimension is a basic independent physical quantity (base quantity) in terms of which other physical quantities may be defined. For example, length is a basic dimension that represents how long an object is. The dimension of length is represented by the symbol [L]. Consequently, Area (L × L) is a physical quantity dimension of [L] × [L].

- Physical quantities can be divided into two categories i.e.
  (a) Those with dimensions e.g. Length, Mass, Time, Area etc.
  (b) Those without dimensions e.g. relative density

- The dimensions (and hence the units) of all other physical quantities (called *derived units* - see Table 1-2) can be constructed (derived) from the base dimensions (units) using *dimensional analysis method*.

Table 1-1: Base quantities

Base quantity	***Name of unit***	***Abbreviation of unit***	***Dimension***
Length	Metre	M	[L]
Mass	Kilogram	Kg	[M]
Time	Second	S	[T]
Electric current	Ampere	A	[A]
Temperature	Kelvin	K	[θ]
Amount of Substance	Mole	Mol	-
Luminous Intensity	Candela	Cd	-

Table 1-2: Some derived quantities

Derived quantity	***Name of unit***	***Other form of unit***	***Base units***	***Dimension***
Frequency	Hertz (Hz)	-	s^{-1}	$[T]^{-1}$
Force	Newton (N)	-	$Kgms^{-2}$	MLT^{-2}
Energy	Joule (J)	Nm	Kgm^2s^{-2}	ML^2T^{-2}
Power	Watt (W)	Js^{-1}	Kgm^2s^{-3}	ML^2T^{-3}
Pressure	Pascal (Pa)	Nm^{-2}	$Kgm^{-1}s^{-2}$	$ML^{-1}T^{-2}$
Electric Charge	Coulomb (C)	-	As	-
Electric p.d.	Volt (V)	JC^{-1}	$Kgm^2A^{-1}s^{-3}$	$ML^2A^{-1}T^{-3}$
Magnetic flux	Weber W)	Tm^2	$Kgm^2A^{-1}s^{-2}$	$ML^2A^{-1}T^{-2}$

1.1.3 Dimensional Analysis (balancing units)

- By the dimensions of a physical quantity, we mean the way it is related to the base quantities (i.e., mass, length and time). For example Area (length × breath) has the dimensions of (L × L) or L^2 with units of m^2.

- Quantities with the same dimensions can be added, subtracted or equaled e.g. in the equation: $W = A + C$; $\Rightarrow$ W, A and C must have the same dimensions. "*You can't add apples to cows*".

- Generally, in scientific equations, the dimensions and hence the units on each side of the equation must be the same (*dimensional homogeneity*). For example, consider the equation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where T is the periodic time of a simple pendulum, l is its length and g is the acceleration due to gravity. By equating the dimensions on both sides we have

$$[T] = \{[L][L]^{-1}[T]^{2}\}^{1/2}$$

$$\text{Thus} \quad [T] = [T]$$

NB. This analysis has made no reference to S.I. units and is applicable to whatever system. Checking equations by balancing their base units is termed the *method of dimensional analysis*.

- Dimensional analysis can assist in
  (a) Testing the correctness of equations and assisting recall of important formulae
  (b) Helping solve physical problems theoretically
  (c) Suggesting relationships between fundamental constants.

Worked examples

1. In the gas equation, $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, find the dimensions of the constants a and b.

   Solution.
   P represents pressure and V the volume. Since a/V^2 is added to P, then both must have the same dimensions. The dimensions of P and V are

$$P = \frac{Force}{Area} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2} \quad \text{and } V = L^3$$

$$\text{Thus} \quad \frac{[a]}{V^2} = \frac{[a]}{L^6} = ML^{-1}T^{-2} \quad \Rightarrow \quad [a] = ML^5T^{-2}$$

Likewise, $[b]$ must rep vol since it is subtracted from V. $\Rightarrow$ $[b] = L^3$.

2. In a simple pendulum experiment, the period $\Im$ depends only on mass m of the bob, the length l of the string and the acceleration $\boldsymbol{g}$, of free fall. Using the method of dimensional analysis, determine the relationship between the period in terms of l and $\boldsymbol{g}$.

   Solution

Let us assume the relationship of the form $\Im = kM^xL^yg^z$ **……….(a)**

where x, y, z and k are unknowns.

The dimensions of g, m, l and $\Im$ are $g = LT^{-2}$, $m = M$, $l = L$, and $\Im = T$

Substituting for these in (a) we have

$$T = M^xL^y(LT^{-2})^z$$ **...……………………………………..……..(b)**

The dimensions of both sides of (b) should be the same. By equating the indices of M, L and T on both sides of Eqn. (b) we get

$x = 0$, $y + z = 0$ and $2z = 1$; solving gives $z = -½$, $y = ½$ and $x = 0$

Substituting for these values in (a)

$$\Rightarrow \qquad T = kL^{\frac{1}{2}}g^{-\frac{1}{2}} = k\sqrt{\frac{l}{g}}$$

k is a constant which cannot be determined by method of analysis but can be found experimentally.

TUTORIAL 1.1

1. A standing wave is set up in a stretched string by plucking it. The velocity ($\boldsymbol{v}$) of the wave in the plucked string depends on the tension ($\boldsymbol{F}$) in the string, its length l and its mass m. By using the method of dimensions, express the velocity $\boldsymbol{v}$ in terms of the tension $\boldsymbol{F}$, the length l and mass (m) of the string.

2. A sphere of radius a moving through a fluid of density ρ with velocity $\boldsymbol{v}$ experiences a retarding force F given by $F = K.a^xP^yV^z$, where K is a non-dimensional coefficient. Use the method of dimensions to find the values of x, y and z.

3. The coefficient of viscosity of a liquid is $\eta = \dfrac{Frictional\ force}{Area \times Velocity\ \ gradient}$. Using the method of dimensional analysis, determine the basic units of η.

1.2 Scalars and Vector Quantities

Assessment Objectives

- *Candidates should be able to*

(i) *Distinguish between vector and scalar quantities and give examples*
(ii) *Add and subtract coplanar vectors*
(iii) *Resolve vectors into its respective components*

Dimensional physical quantities such as length, mass, time, temperature etc be described completely by a single real number i.e. magnitude only (the "how much" or "how big" part) with the units of measurement. Such quantities are termed *scalars* and can be represented analytically by a letter such as *t*, *m*, etc.

On the other hand, quantities such as displacement (***s***), velocity (***v***), acceleration (***a***), force (***F***) etc. have direction associated with them and such quantities (specified by both magnitude and direction) are termed *vectors*. For example, the motion of an airplane is completely described by how fast and in what direction it is moving.

- A vector can be represented analytically by a letter with an arrow above it e.g., ($\vec{A}$), a bold-faced letter, **A** (see Fig. 1-1) or geometrically by an arrow *PQ* where *P* is the *initial* point and *Q* is the *terminal* point. The *magnitude* or length of a vector **A** is itself a scalar and is denoted by $|\mathbf{A}|$.

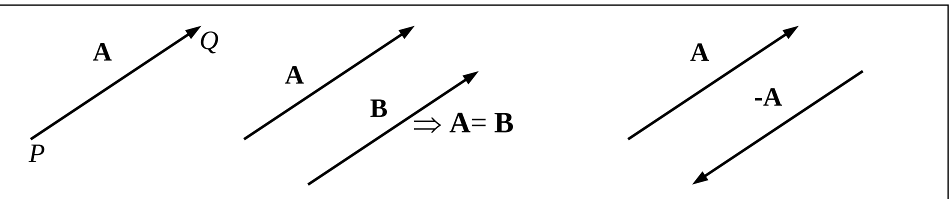


Fig. 1-1 Fig. 1-2 Fig. 1-3

1.2.1 Vector Algebra

1. Two vectors **A** and **B** are *equal* if they have the same magnitude and direction regardless of their initial points. Thus **A** = **B** in Fig. 1-2 above.

2. A vector of same length as **A** (Fig. 1-3) but of opposite direction, is denoted as –**A**.

3. The *sum* or *resultant* of two vectors **A** and **B** is a vector **C** obtained following the *parallelogram law* (by completing a parallelogram) i.e. **C** is obtained by placing the

initial point of **B** to the terminal point of **A** and joining the initial point of **A** to the terminal point of **B** (Fig. 1-4) or vice versa.

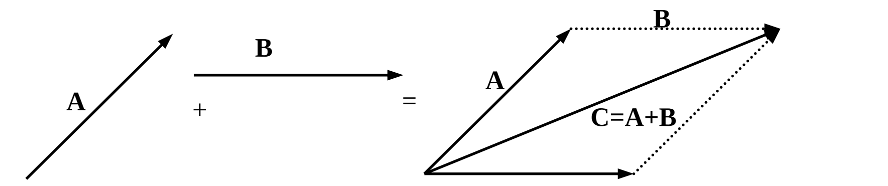


Fig. 1-4

4. The product of a vector **A** by a scalar p is a vector p**A** or **A**p with magnitude $|p|$ times the magnitude of **A** and direction the same as or opposite to that of **A** depending on the sign of p. If p = 0, then p**A** = 0, the *null vector*.

1.2.2 Laws of Vector Algebra

- If **A**, **B** and **C** are vectors, and p and q are scalars, then

1. **A + B** = **B + A** — commutative law of addition is obeyed
2. **A +** (**B** + **C**) = (**A + B**) + **C** — Associative law of Addition
3. p(q**A) =** (pq)**A** = q(p**A**) — Associative law of multiplication
4. (p + q)**A =** p**A** + q**A** — Distributive law
5. p(**A** + **B**) **=** p**A** + p**B** — Distributive law

1.2.3 Co-ordinate System

Vectors can be represented in two or three-dimensional Cartesian Co-ordinate systems. Fig 1-5 shows a right-handed 3-D Cartesian co-ordinate system where the y and z axes are taken in the plane of the paper while the x-axis is perpendicular to the plane of the paper pointing outwards. A left-handed system is a mirror reflection of a right-handed system i.e., with the x-axis perpendicular to the plane of the paper but pointing inwards.

1.2.4 Vector Components (resolution of vectors)

- Vectors can be added or subtracted easily by making use of the respective components. Generally, a vector **A** represented in a 3-dimensional rectangular coordinate system [Fig.1-6], will have the *component vectors* $A_x\boldsymbol{i}$, $A_y\boldsymbol{j}$ and $A_z\boldsymbol{k}$ where, A_x, A_y and A_z are the rectangular coordinates/components of the terminal point of **A** in the x, y and z directions respectively.

- ***i***, ***j*** and ***k*** are mutually perpendicular unit vectors of magnitude 1 and have the directions of the positive x, y and z-axes respectively [Fig. 1-5].

- From Fig. 1-6, **A** = $A_x\boldsymbol{i}$ + $A_y\boldsymbol{j}$ + $A_z\boldsymbol{k}$. It follows from the parallelogram law that $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

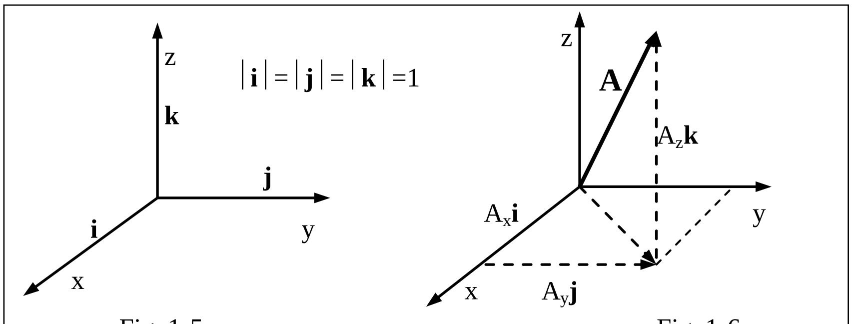


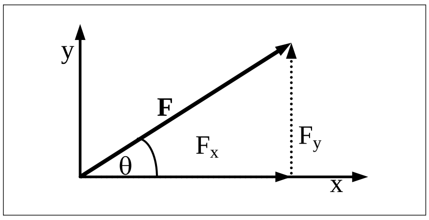
Fig. 1-5 Fig. 1-6

- *Collinear /Parallel vectors* : Are vectors acting along the same or parallel lines
- *Coplanar Vectors*: Are vectors confined to the same plane.

Worked Examples

1. A force ***F*** acts at an angle θ to the horizontal. Find the vertical and horizontal components of ***F***.

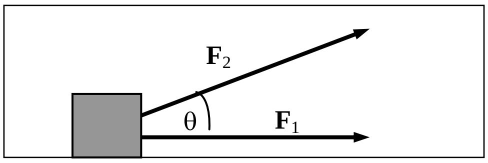
Solution



Sin θ = Opp/hypotenuse = $F_y/\boldsymbol{F}$
Cos θ = Adj/hypotenuse = $F_x/\boldsymbol{F}$
Tan θ =opp/adj = F_y/F_x

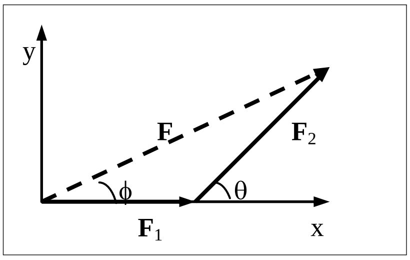
Thus: Horizontal component of **F** = F_x =**F**cos θ
Vertical component of **F** = F_y =***F***sin θ

2. *Two forces* $\mathbf{F_1}$ *and* $\mathbf{F_2}$ *act on an object as shown in the figure below. Determine the magnitude of the resultant force.*



Solution

By resolving the forces into their respective components we have



$F_{1x} = F_1$
$F_{2x} = F_2\cos\theta$
$F_{1y} = 0$
$F_{2y} = F_2\sin\theta$
Thus
$F_x = F\cos\phi = F_{1x} + F_{2x} = F_1 + F_2\cos\theta$
$F_y = F\sin\phi = F_{1y} + F_{2y} = F_2\sin\theta$

Hence $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} = (F_1x + F_2x)\mathbf{i} + (F_1y + F_2y)\mathbf{j}$.

$$\Rightarrow \qquad |F| = \sqrt{F_x^2 + F_y^2} = \sqrt{(F_{1x} + F_{2x})^2 + (F_{1y} + F_{2y})^2}$$

$$\Rightarrow \qquad |F| = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

1.3 Scalar and Vector Multiplication

(a) Scalar or Dot product

- The dot product (.) of any two vectors **A** and **B** (i.e. **A.B**) is a product of the magnitudes of **A** and the component of **B** parallel to **A** (see Fig. 1-7) i.e.

$\mathbf{A.B} = AB\cos\theta = BA\cos\theta,\ 0 \le \theta \le \pi$ NB. **A.B** is a scalar

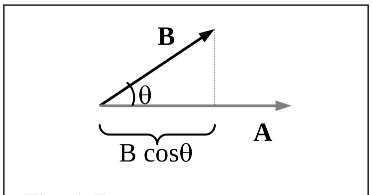


Fig. 1-7

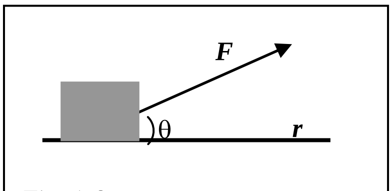


Fig. 1-8

- Thus a dot product is basically the product of two or more vectors (or their components) that are in the same direction. For example, if a force ***F*** acting on a body at an angle θ, then the work (W) done over a displacement ***r*** can be given by (see Fig.1-8)

W = (component of **F** in the direction of ***r***)(displacement)
= (F$\cos\theta$) ***r***
= ***F.r*** or ***r.F***

Rules/properties of dot product

- If **A**, **B** and **C** are vectors, and *p* and *q* are scalars, then

1. **A.B** = **B.A** commutative law of dot product
2. **A.**(**B** + **C**) = (**A.B**) + (**A.C**) Distributive law
3. *p*(**A.B**) = (*p***A**)**.B** = **A.**(*p***B**) = (**A.B**)*p*
4. **i.i** = **j.j** = **k.k** = 1 and **i.j** = **j.k** = **k.i** = 0 since ***i***,***j*** & ***k*** are perpendicular
5. If **A** = $A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}$ and **B** = $B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}$ then:

$$\mathbf{A.B} = A_xB_x + A_yB_y + A_zB_z$$
$$\mathbf{A.A} = A_x^2 + A_y^2 + A_z^2 = \mathbf{A}^2$$
$$\mathbf{B.B} = B_x^2 + B_y^2 + B_z^2 = \mathbf{B}^2$$

6. If **A.B** = 0 and **A** and **B** are not null vectors, then **A** and **B** are perpendicular.

Direction Cosines

- The direction cosines (x, y and z-direction cosines) of a vector **A**, are the cosine of the angles which a vector makes with the x, y and z axes respectively. These are denoted by *l*, *m* and *n* respectively, where

$$l = \cos\alpha = \frac{A_x}{|A|}; \quad m = \cos\beta = \frac{A_y}{|A|}; \quad n = \cos\gamma = \frac{A_z}{|A|}$$

And $l^2 + m^2 + n^2 = 1$

(b) Vector product (Cross product)

- The cross (x) product of two vectors **A** and **B** is a vector **C** which is perpendicular to the plane containing **A** and **B** in a direction determined by the *right-hand rule* (Fig. 1-9) i.e., the direction of **C** is the direction in which a *right-hand-screw* advances if turned from **A** towards **B**. **A**, **B** and **C** form a right-handed system. Thus:

$$\mathbf{A} \times \mathbf{B} = AB\sin\theta\,\mathbf{u} = \mathbf{C}, \quad 0 \le \theta \le \pi$$

where **u** is a unit vector indicating the direction of **A × B.**.

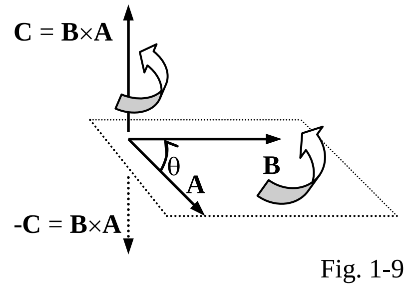


Fig. 1-9

Cross product will be used to describe Torque, Angular momentum and extensively for magnetic fields.

Rules of cross product

- If **A**, **B** and **C** are vectors, and p and q are scalars, then

1. **A × B** = **-B × A** Commutative law of cross pdt fails
2. **A** ×(**B + C**) = (**A × B**) + (**A × C**) Distributive law
3. p(**A × B**) = (p**A**) × **B** = **A** × (p**B**) = (**A × B**)p
4. **i × i** = **j × j** = **k × k** = 0 and **i × j** = **k**, **j × k** = **i**, **k × i** = **j** [see Fig.1-5]
5. If **A** = $A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}$ and **B** = $B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}$ then:

$$A \times B = \begin{bmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

$$= \left(A_y B_z - A_z B_y\right) i + \left(A_z B_x - A_x B_z\right) j + \left(A_x B_y - A_y B_x\right) k$$

6. If **A = B** or if **A** is parallel to **B,** then **A × B =** 0.

1.3.1 Derivatives of Vectors

- If a vector **A** is a (vector) function of a scalar variable u, i.e. **A**(u), then the derivative of **A**(u) is defined as

$$\frac{d\mathbf{A}}{du} = \lim_{\Delta u \to 0} \frac{\mathbf{A}(u + \Delta u) - \mathbf{A}(u)}{\Delta u}$$

Thus: if **A**(u) = $A_x(u)\boldsymbol{i} + A_y(u)\boldsymbol{j} + A_z(u)\boldsymbol{k}$ then;

$$\frac{d\mathbf{A}}{du} = \frac{dA_x}{du}\mathbf{i} + \frac{dA_y}{du}\mathbf{j} + \frac{dA_z}{du}\mathbf{k}$$

- For example, If **A** = $(2u^2 - 3u)\boldsymbol{i} + (5 \cos u)\boldsymbol{j} - (3 \sin u)\boldsymbol{k}$, then

$$\frac{d\mathbf{A}}{du} = (4u - 3)\mathbf{i} - 5\sin u\mathbf{j} - 3\cos u\mathbf{k}; \quad \frac{d^2\mathbf{A}}{du^2} = 4\mathbf{i} - 5\cos u\mathbf{j} + 3\sin u\mathbf{k}$$

1.3.2 Integrals of Vectors

- If **A**(u)= $A_x(u)\boldsymbol{i} + A_y(u)\boldsymbol{j} + A_z(u)\boldsymbol{k}$ is a vector function of u, then the *indefinite integral* of **A**(u) is defined as

$$\int \mathbf{A}(u)du = \mathbf{i}\int A_x(u)du + \mathbf{j}\int A_y(u)du + \mathbf{k}\int A_z(u)du$$

If there exists a vector function **B**(u) such that **A**(u) = $\frac{d}{du}\{\mathbf{B}(u)\}$, then

$$\int \mathbf{A}(u)du = \int \frac{d}{du}\{\mathbf{B}(u)\} = \mathbf{B}(u) + C$$

where C is an arbitrary constant vector independent of *u*. The *definite integral* between limits $u = \alpha$ and $u = \beta$ is, in a such case, given by

$$\int_{\alpha}^{\beta} \mathbf{A}(u)du = \int_{\alpha}^{\beta} \frac{d}{du}\{\mathbf{B}(u)\}du = \mathbf{B}(u) + c\Big|_{\alpha}^{\beta} = \mathbf{B}(\beta) - \mathbf{B}(\alpha)$$

- For example, If $\mathbf{A} = (2u^2 - 3u)\boldsymbol{i} + (5\cos u)\,\boldsymbol{j} - (3\sin u)\,\boldsymbol{k}$, then

$$\int \mathbf{A}(u)du = \left(\frac{2}{3}u^3 - \frac{3}{2}u^2\right)\mathbf{i} - 5\sin u\mathbf{j} - 3\cos u\mathbf{k} + C$$

The definite integral can also be defined as a limit of a sum analogous to that of elementary calculus. For more on vector derivatives and integrals, please refer to calculus.

Assignment 1.1

1. Which line in the table correctly identifies Force, Kinetic energy and Momentum as scalar or vector quantities?

	Force	*Kinetic energy*	*Momentum*
(a)	Scalar	Scalar	Scalar
(b)	Scalar	Vector	Vector
(c)	Vector	Scalar	Scalar
(d)	Vector	Scalar	Vector
(e)	Vector	Vector	Vector

TUTORIAL 1.2

1. If $\boldsymbol{a} = 4\boldsymbol{i} - 3\boldsymbol{j}$ and $\boldsymbol{b} = 6\boldsymbol{i} + 8\boldsymbol{j}$, find the magnitudes of
(i) ***a*** (ii) ***a*** + ***b*** and (iii) ***b*** - ***a***

2. A body of mass 1.5Kg is placed on a plane surface inclined at 30^o to the horizontal. Calculate the friction and the normal reaction forces which the plane must exert if the body is to remain at rest ($g = 10ms^{-2}$).

3. Two vectors ***a*** and ***b*** have equal magnitudes of 10 units each and are oriented as shown in Fig. 1. If their vector sum is ***r***, determine
(i) the x and y components of ***r***
(ii) the angle that r makes with the x axis

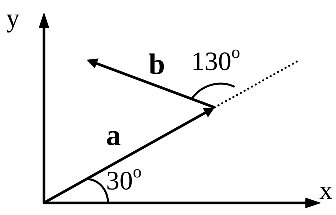


Fig.1

4. A man is dragging a log along a horizontal surface by means of a rope. The rope makes an angle of 30° to the horizontal, and the tension in the rope is 400N. Calculate the components of the tension along the horizontal and the vertical directions.

5. Show that **A**.(**B** + **C**) = **A.B** + **A.C**

6. Raindrops of mass 5×10^{-7} Kg falls vertically in still air with a uniform speed of 3 ms^{-1}. If such drops are falling when a wind is blowing with a speed of 2 ms^{-1}, what is the angle that the path of the drops makes with the vertical?. Determine the kinetic energy of the drops.

7. If vector ***A*** = ***i*** + 2***j*** - 2***k*** , ***B*** = 2***i*** + ***j*** + ***k*** and ***C*** = ***i*** - 3***j*** - 2***k*** find the magnitude and direction cosines of the vector ***R*** = ***A*** + ***B*** + ***C***.

8. Show that the vectors ***A*** =3***i*** - 2***j*** + ***k*** , ***B*** = ***i*** -3***j*** + 5***k*** and ***C*** = 2***i*** + ***j*** - 4***k*** form a right handed triangle.

1.4 Rectilinear Motion

The mathematical concept of vectors is useful for the description of displacement, velocity and acceleration in one, two or three-dimensional motion. A body can undergo either one or a combination of the following motions, namely;

(i) Translational or Rectilinear motion i.e. motion in a straight line or one-dimensional motion
(ii) Rotational or Circular motion e.g. a rotating wheel
(iii) Vibrational or Oscillatory motion e.g., a pendulum clock.

Assessment Objectives

- *At the end of this section, candidates should be able to*

(j) Define displacement, speed, velocity and acceleration
(v) Use graphical methods to represent displacement, speed, velocity and acceleration
(vi) Find the distance traveled by calculating the area under a velocity-time graph
(vii) Evaluate the velocity from displacement-time graphs and acceleration from velocity-time graphs
(viii) Derive and apply the equations of motion for uniformly accelerated objects
(ix) Describe qualitatively motion of bodies falling in a uniform gravitational field

In this chapter, we address ourselves to defining the terms associated with rectilinear motion. For simplicity, consider the motion of a car which begins from rest, increases its speed for a time before being brought to rest. Fig. 1-10 shows the distance time graph of the car. From the figure, we can define the following parameters

Displacement (s)

- It is a *vector* that specifies the position of an object relative to the initial point (origin) or *it is the distance moved in a given direction* e.g., we can say the car has moved 20Km due north.

Velocity (v)

- *The velocity* (**v**) *of an object is a vector and is defined as the rate of change of displacement* i.e.,

$$Average\ v = \frac{change\ in\ displacement\ (\Delta s)}{time\ taken\ (\Delta t)} = \frac{\Delta s}{\Delta t} \qquad \text{…………..…(1.1)}$$

For example, if $S_2 - S_1$ is the change in position occurring in time interval $t_2 - t_1$, (Fig. 1-10) then:

$$Average\ v = \frac{\Delta s}{\Delta t} = \frac{S_2 - S_1}{t_2 - t_1} = \text{slope of the straight line } P_1 P_2$$

- S*peed is a* ***scalar*** *that measures the rate of change of distance* and is equal to the magnitude of velocity e.g., the average speed (υ) between times t_1 and t_2 in Fig. 1-10 is

$$\upsilon = \frac{Actual\ distance}{time\ taken} = \frac{Curved\ dist\ P_1P_2}{\Delta t} = \left|\frac{\Delta S}{\Delta t}\right| = |\mathbf{v}| \qquad \text{……………(1.2)}$$

where ΔS in this case is the arc length P_1P_2.

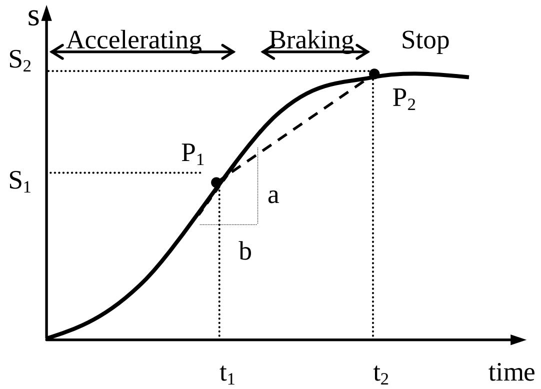


Fig. 1-10. Displacement - time graph of an automobile

1.4.1 Relative Motion

Velocity is a relative quantity. Usually, we talk of velocities relative to the Earth's surface e.g., for a car moving at 100 km/h, it is understood to mean relative to the Earth's surface. If two objects are moving independently, then we may compare the velocity of one object in relation to the other i.e., assuming the other object is virtually at rest.

For example, suppose a train is travelling at **V** *m/s* and a man walks at a speed **v** *m/s* in the train, towards the back of the train. In this case, the velocity of the man relative to the train is **v** *m/s* while his velocity relative to the ground is (**V** - **v**) *m/s*.

Activity

What is the man's relative velocity to the ground if he walked across the train? Represent your argument using a vector diagram.

Note: This topic introduces you to the Theory of Relativity which you will meet in later years.

1.4.2 Acceleration (a)

It is a vector specifying how fast the velocity of an object changes with time. Average acceleration is given by

$$Average\ \mathbf{a} = \frac{change\ in\ velocity\ (\Delta \mathbf{v})}{time\ taken\ (\Delta t)} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.3)$$

Units: displacement (m); speed (ms^{-1}); velocity (ms^{-1}); acceleration (ms^{-2})

The instantaneous velocity (i.e. velocity of an object at a given time or point) is the time rate of displacement i.e.

$$Inst.vel = \frac{d\mathbf{s}}{dt} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.4)$$

For example, if the position vector of the automobile at point P_1 (Fig. 1-9) is $\boldsymbol{r} = \boldsymbol{r}(t) = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, then the instantaneous velocity, acceleration and speed of the automobile at P_1 are

$$Inst.\ \mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$$

$$Inst.\ \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k}$$

$$\text{Speed } \upsilon = |\mathbf{v}| = \left|\frac{d\mathbf{r}}{dt}\right| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} = \frac{d\mathbf{s}}{dt}$$

1.4.3 Time Derivatives

We shall sometimes find it convenient to use dots (**.**) placed over a symbol to denote derivatives with respect to (*w.r.t*) time *t*. One dot for a first derivative, two for a second derivatives etc. For example if ***r*** = ***r****(t)* = *x****i*** +*y****j*** +*z****k*** then

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k}$$

1.4.4 Types of Rectilinear Motion

Among the most common types of rectilinear motion are the motion with uniform velocity and the motion with uniform acceleration.

(i) Motion with uniform velocity

Linear motion can be depicted using ***v***-*t* and displacement/time graphs

Figure 1-11 shows the ***s***-*t* and ***v***-*t* graphs respectively for a body moving with uniform velocity (i.e. constant speed in a fixed direction). Figure 1-12 shows the motion of a ball thrown vertically upwards. ***NB***. since velocity and displacement are vectors, care must be taken with the sign convention for direction (i.e. +ve *y* for upward direction)

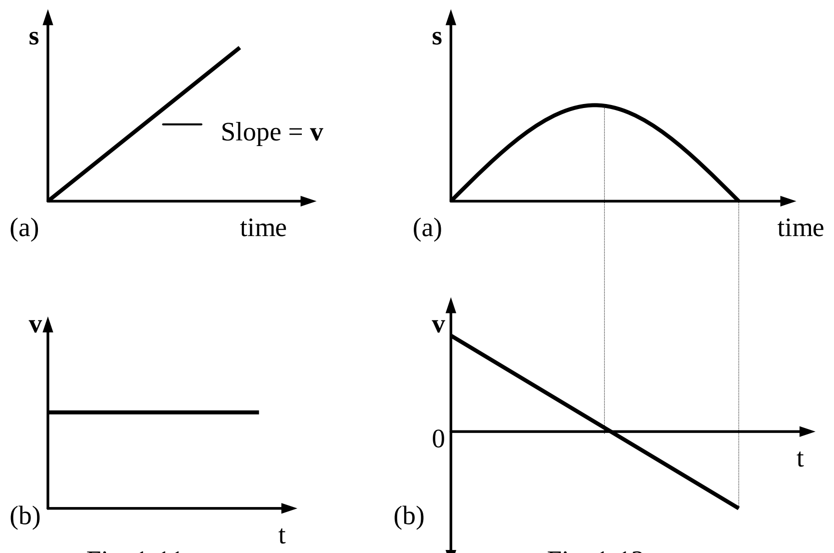


Fig. 1-11

Fig. 1-12

Displacement -Time graph
Slope = Velocity of object
Area has no physical significance

Velocity-time graph
Slope = acceleration of object
Area = Displacement of object

(ii) Motion with Uniform Acceleration (Equations of Motion)

Figures 1-13 and 1-14 show the ***v***-*t* and ***a***-*t* graphs respectively of a uniformly accelerating object. If the velocity of a uniformly accelerating object increases from a value **u** to **v** (Fig. 1-13) in time t, then from the definition of acceleration $\left(a = \frac{v-u}{t}\right)$, we have;

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.5)$$

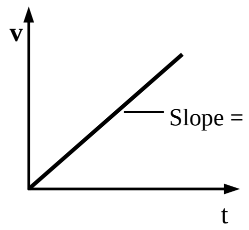


Fig. 1-13 uniform acceleration

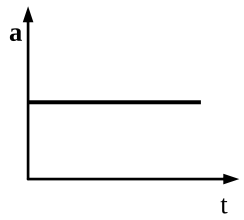


Fig. 1-14 Uniform acceleration

The displacement covered in time t can be given by;

$$\mathbf{s} = (\text{Av. velocity})(\text{time}) = \tfrac{1}{2}(\boldsymbol{u} + \boldsymbol{v})\, t \qquad \ldots\ldots\ldots\ldots (1.6)$$

But $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$$\Rightarrow \qquad \boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\, \boldsymbol{a}t^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1.7)$$

By eliminating t in either equation (1.6) or (1.7) by substituting for $t = (\boldsymbol{v} - \boldsymbol{u})/\boldsymbol{a}$ (from Eqn. 1.5) we have

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.8)$$

Equation (1.5), (1.7) and (1.8) are the equations of motion for an object moving in a straight line with uniform acceleration.

Note: The displacement covered by a uniformly accelerating object can also be given as the total area under the velocity-time graph (see Fig. 1-15)

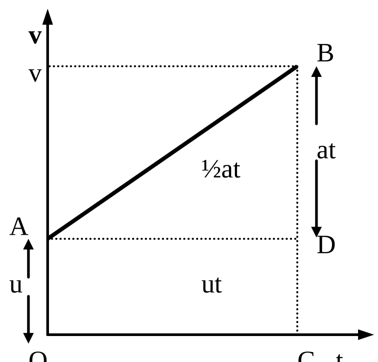


Fig. 1-15

The area under the **v**-t graph
= shaded area OABC
= Area DABD + Area OADC
= ½t(BD) + ut

But BD = increase in velocity in time t
= $\boldsymbol{v}$-$\boldsymbol{u}$ = $\boldsymbol{a}t$

$\Rightarrow$ Total area = $\tfrac{1}{2}t\,(\boldsymbol{a}t) + \boldsymbol{u}t$
= $\boldsymbol{u}t + \tfrac{1}{2}\,\boldsymbol{a}t^2$

Thus $\boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\,\boldsymbol{a}t^2$

Worked examples

1. A body covers a distance of 10m in 4s, it rests for 10s and finally covers a distance of 90m in 6s. Calculate its average speed.

   Solution

   Total distance = (100)m; Total time = (20)s; $\Rightarrow$ Av. speed = $5ms^{-1}$

2. A man runs 800m due north in 110s, followed by 400m due south in 90s. Calculate his average speed and his average velocity for the whole journey.

   Solution

   (i) Av. speed = (total dist./total time) = (800 + 400)m/(200)s = $6ms^{-1}$

   (ii) Av. velocity = (disp/time) = (800-400)m/(200)s = $2ms^{-1}$ due North.

3. A car moving with a velocity of 54Km/h accelerates uniformly at the rate of $2ms^{-2}$. Calculate the distance traveled from the place where the acceleration begun to that where the velocity reaches 72Km/h and the time taken to cover this distance.

Solution

Given 54 km/h =$15ms^{-2}$; 72 km/h =$20ms^{-2}$ and $a = 2ms^{-2}$

(i) Using $\boldsymbol{v^2 = u^2 + 2as} \Rightarrow$ $20^2 = 15^2 + 2(2s)$ or s = 43¾m

(ii) Using $\boldsymbol{v = u + at}$ $\Rightarrow$ 20 = 15+2t or t = 2.5s

4. A particle moves along a curve whose parametric equations *are* $x = 3e^{-2t}$, $y = 4\sin 3t$, $z = 5\cos 3t$ where t is the time. Find the velocity and acceleration of the particle at any time t.

   Solution

   Position vector of the particle is $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = 3e^{-2t}\mathbf{i} + 4\sin 3t\mathbf{j} + 5\cos 3t\mathbf{k}$

$$\Rightarrow \quad v = \frac{dr}{dt} = -6e^{-2t}\mathbf{i} + 12\cos 3t\mathbf{j} - 15\sin 3t\mathbf{k}$$

$$\Rightarrow \quad a = \frac{dv}{dt} = 12e^{-2t}\mathbf{i} - 36\sin 3t\mathbf{j} - 45\cos 3t\mathbf{k}$$

Questions for discussion

1. Give examples to answer the following questions
   (a) Can a body have a constant velocity and varying speed?
   (b) Can the direction of a body's velocity change when it has constant acceleration?
2. A ball is thrown vertically upwards from a trolley that is moving horizontally with constant velocity. Describe the path of the ball as seen by an observer (a) on the trolley, and (b) on the ground near the trolley.

TUTORIAL 1.3

1. A body moves 3000m due east in 40s and then 400m due north in 60s. Calculate its average velocity for the whole journey and its direction.

2. A car decelerates uniformly from a velocity of 10ms-1 to rest in 2s. If it takes 2s to reverse with uniform acceleration to its original position,
   (i) plot a v-t and a speed-time graph for the whole journey
   (ii) determine (a) the displacement covered and the average velocity of the car; (b) the distance covered and the average speed of the car

3. Starting from rest, a car travels for 2 min with a uniform acceleration of 0.3 m s^{-2}, after which the velocity is kept constant for 4000 m. The car is then brought to rest with a uniform retardation 0.6 ms^{-2}. What is the time taken for the journey?

4. A ship travels due east at 3.0 ms^{-1}. If it now heads due north at the same speed, determine the change in velocity.

5. A car takes 80s to travel at constant speed in a semicircle from point A to B as shown in Fig. 2. Calculate
   (i) its speed
   (ii) its average velocity and
   (iii) the change in velocity from point A to B.

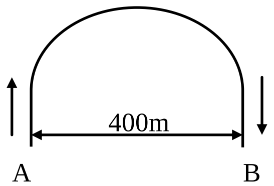


Fig. 2.

6. How far does a body travel in the fourth second if it starts from rest with a uniform acceleration of 2.0 ms^{-2}?

7. A particle moves along a curve whose parametric equations *are* $x = 3e^{-2t}$, $y = 4 \sin 3t$, $z = 5 \cos 3t$ where t is the time. Find
   (i) the velocity and acceleration of the particle at any time t
   (ii) the magnitudes of the velocity and acceleration at $t = 0$.

8. A particle travels so that its acceleration is given by $a = 2e^{-t}\mathbf{i} + 5 \cos t\mathbf{j} - 3 \sin t\mathbf{k}$. If the particle is located at (1, -3, 2) at time $t = 0$ and is moving with a velocity given by $4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, find the velocity and the displacement of the particle at any time $t > 0$.

9. Two particles have position vectors given by $\mathbf{r}_1 = 2t\mathbf{i} - t^2\mathbf{j} + (3t^2 - 4t)\mathbf{k}$ and $\mathbf{r}_2 = (5t^2 - 12t + 4)\mathbf{i} + t^3\mathbf{j} - 3t\mathbf{k}$. Find
   (i) the relative velocity and
   (ii) the relative acceleration of the second particle with respect to the first at the instant where $t = 2$.

10. Rain is falling vertically at 8.0ms^{-1} relative to the ground. The raindrops make tracks on the side window of a car at an angle of 30^o below the horizontal. Calculate the speed of the car.

1.5 Motion in a Uniform Field

A force field is any region in which an object experiences a force while lying in that region. For example, the region above the Earth's surface in which the force of gravity is present can be termed a *gravitational force field*.

The gravitational force has a constant magnitude and direction (*a uniform force field*. If a particle of constant mass moves in a uniform force field, then it experiences a uniform acceleration.

1.5.1 Free Falling Bodies

- Near the earth's surface, objects fall towards the earth with a uniform acceleration (g = 9.8ms^{-2}) provided air resistance is negligible. Such objects are said to be in *free fall*. The value of g is the same for all bodies released at same location and is independent of the body's speed, mass, size or shape.

- The equations of motion for a freely falling body (Eqns. 1.9 - 1.11) are similar to those for linear motion (with uniform acceleration) but with $\boldsymbol{a}$ being replaced by $\boldsymbol{g}$. For upward motion (a rising object), $\boldsymbol{g}$ is –ve since the body decelerates (see Fig. 1-15). If the body falls from rest then $\boldsymbol{u}$ = 0.

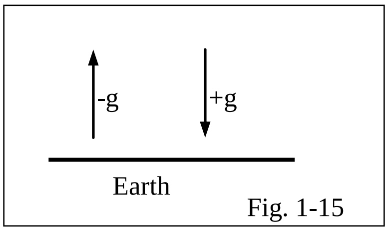


Fig. 1-15

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{g}t \qquad \ldots\ldots\ldots\ldots\ldots(1.9)$$

$$\boldsymbol{s} = \boldsymbol{u}t + \tfrac{1}{2}\boldsymbol{g}t^2 \qquad \ldots\ldots\ldots\ldots\ldots(1.10)$$

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{g}\boldsymbol{s} \qquad \ldots\ldots\ldots\ldots\ldots(1.11)$$

Worked example

5. A cricket ball is thrown vertically upwards with a velocity of 20.0ms^{-1}. Neglecting air resistance, calculate
   (i) the maximum height reached
   (ii) the time taken to return to the ground.

Solution

Taking upward direction as positive, $\boldsymbol{u}$ = 20 ms^{-1} and $\boldsymbol{a} = -\boldsymbol{g}$ = -10 ms^{-2}

(i) At maximum height, $\boldsymbol{v}$ = 0 ms^{-1}, thus from $\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{a}\boldsymbol{s}$

$\Rightarrow \quad 0 = (20)(20) + 2(-10)\,\boldsymbol{s}; \quad \therefore\ \boldsymbol{s} = 20\text{m}$

(ii) On return to ground, $\boldsymbol{s}$ goes to zero, thus using $\boldsymbol{s} = \boldsymbol{u}\boldsymbol{t} + \tfrac{1}{2}\boldsymbol{a}\boldsymbol{t}^2$

$\Rightarrow \quad 0 = 20t + \tfrac{1}{2}(-10)t^2 \quad \therefore t = 0\text{s or } 4\text{s}$

1.5.2 Projectile Motion

A projectile is any object launched at an angle such that as it moves forward, it also moves upwards/downwards under the influence of gravity (Fig. 1-16) and as such, the motion is a non-linear (2-D in x-y plane). Examples include a ball kicked by a football player or a mortar fired from a rocket (gun).

- If air resistance is neglected, then a projectile can be considered as a freely falling object and its equations of motion determined from the linear equations of motion together with the initial conditions. If the coordinate system is chosen with the +ve y-axis vertically upwards, then the initial velocity has two components, $\boldsymbol{u}\cos\theta$ along the horizontal and $\boldsymbol{u}\sin\theta$ along the vertical directions respectively. Likewise, the vertical and horizontal component of the acceleration are $\boldsymbol{a}_y = -g$ and $\boldsymbol{a}_x = 0$. The horizontal and vertical motions as analyzed as shown in Fig. 1-16.

Worked Example

1. A projectile is launched with an initial speed of $\boldsymbol{u}$ ms^{-1} and at an angle θ to the horizontal. Determine
   (i) The time it takes to reach the highest point
   (ii) The highest point reached
   (iii) The time of flight back to earth
   (iv) The range
   (v) Prove that the range of the projectile is a maximum when angle $\theta = 45^o$
   (vi) Show that the path of a projectile is parabolic.

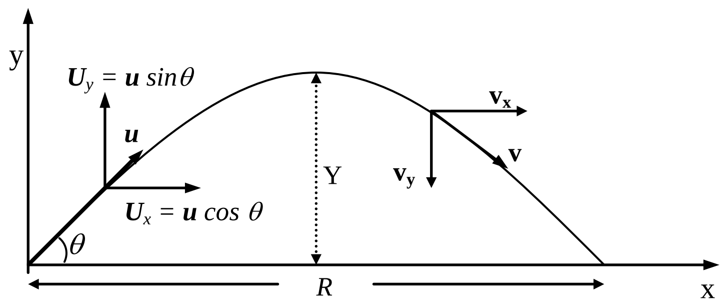


Fig. 1-16

Horizontal motion		**Vertical motion**	
Initial velocity	$= \boldsymbol{u}\cos\theta$	Initial velocity	$= \boldsymbol{u}\sin\theta$
Acceleration ($\boldsymbol{a}_x$)	$= 0$	Acceleration ($\boldsymbol{a}_y$)	$= -g$ (-ve for upward)
Dist. (Range), R	$= \boldsymbol{u}\cos\theta\, t$	Distance moved, Y	$= \boldsymbol{u}\sin\theta\, t - ½ gt^2$
Final speed $\boldsymbol{V}_x$	$= \boldsymbol{u}\cos\theta$ (i.e.,constant)	Final speed $\boldsymbol{V}_y$	$= \boldsymbol{u}\sin\theta - gt$
$\boldsymbol{V}_x^2 = \boldsymbol{u}^2\cos^2\theta$		$\boldsymbol{V}_y^2 = \boldsymbol{u}^2\sin^2\theta - 2gs$	

Solution

(i) Using $\boldsymbol{v} = \boldsymbol{u}\ sin\ \theta$ -gt , at the highest point, vertical component of velocity ($\boldsymbol{v_y}$) is zero.

$$\Rightarrow \quad t = \frac{\mathbf{u}\sin\theta}{g} \qquad \text{.............................(1.12)}$$

(ii) Using $\boldsymbol{Y} = \boldsymbol{u}\ sin\ \theta\, t$ - ½ $\boldsymbol{g}t^2$ where $t = \frac{\mathbf{u}\sin\theta}{g}$

$$\Rightarrow \quad Y = \mathbf{u}\sin\theta\left(\frac{\mathbf{u}\sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{\mathbf{u}\sin\theta}{g}\right)^2$$

$$\Rightarrow \quad Y = \frac{\mathbf{u}^2\sin^2\theta}{2g} \qquad \text{.............................(1.13)}$$

(iii) Time of flight back to earth is when $\boldsymbol{s}$ becomes zero. Thus from $\boldsymbol{s} = \boldsymbol{u}\ sin\ \theta\ t$ - ½ gt^2

$$\Rightarrow \quad 0 = \mathbf{u}\ sin\ \theta\, t - ½\, gt^2 \quad \text{but } t \neq 0$$

$$\Rightarrow \quad t = \frac{2\mathbf{u}\sin\theta}{g} \qquad \text{.............................(1.14)}$$

Note: This is twice the time to reach maximum height

(iv) The range R = (Horizontal component of velocity)(time of flight back to earth)

$$= \boldsymbol{u}\ cos\ \theta\, t \ = \ \mathbf{u}\cos\theta \times \frac{2\mathbf{u}\sin\theta}{g}$$

$$\Rightarrow \quad R = \frac{\mathbf{u}^2\sin 2\theta}{g} \qquad \text{.............................(1.15)}$$

(v) From (iv), R is maximum when sin 2θ = 1. Thus θ = 45°.

(vi) A parabolic path obeys the equation y = bx - cx^2 where b and c are constants. From part (ii) above, the vertical and horizontal displacement y and x are

$$y = \mathbf{u}\ sin\ \theta\, t - ½\, \boldsymbol{g}t^2 \qquad \text{.............................(a)}$$

$$x = \mathbf{u}\ cos\ \theta\, t \qquad \text{.............................(b)}$$

From (b) $\Rightarrow$ $t = \frac{x}{\mathbf{u}\cos\theta}$. substituting for t in (a) gives

$$y = \mathbf{u}\sin\theta\left(\frac{x}{\mathbf{u}\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{\mathbf{u}\cos\theta}\right)^2 = xTan\theta - \frac{g}{2(\mathbf{u}\cos\theta)^2}x^2$$

$\Rightarrow$ $y = bx - cx^2$ Hence the trajectory is parabolic.

Note: The magnitude of the resultant velocity vector (speed) at any instant is $\upsilon = \sqrt{V_x^2 + V_y^2}$. Likewise, the displacement magnitude, **S** after time *t*, is **S** = $= \sqrt{R^2 + Y^2}$. The angle θ that the velocity vector makes with the horizontal at that instant is given by $Tan\theta = \frac{V_x}{V_y}$.

1.6 Circular Motion

Assessment Objectives

- *Candidates should be able to*

(x) *Describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of uniform motion in a circle Express angular displacement in radians*

(xi) *Use the concept of angular velocity*

(xii) *Recall and use $v = rw$, $a = rw^2$, $a = v^2/r$*

Circular motion, like projectile motion is a 2-D motion. The simplest case is where an object moves in a circle with uniform speed (i.e. *uniform circular motion*) e.g., the motion of planets round the sun, electrons orbiting the nucleus, an object on a string being whirled round, a car moving around a traffic circle (roundabout) etc.

Notably, the earth rotates on its axis once a day -the right speed to produce moderate temperatures. If it took longer, the temperature would rise and runaway greenhouse effect (overheating) would occur. What would happen if it the earth rotated faster?

Figure 1-17 shows an object undergoing uniform circular motion (i.e., with constant speed). At any point, e.g. point *A*, the object has a natural tendency (on account of inertia) to move in the direction of *AC* at a tangent to the circle. The tension in the string *T* (towards the centre) overcomes this tendency and causes the body to move in the circular path.

Owing to its inertia, the body resists the pull in the string (i.e., *T*) by means of an equal and opposite force *F* in the direction away from the centre (by Newton's 3rd law). *T* is thus the *Centripetal force* (tending towards the centre) while *F* is the *Centrifugal force* (tendency to fly away from centre)

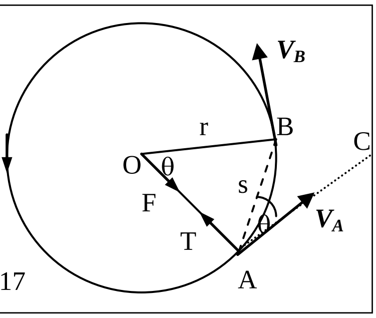


Fig. 1.17

NB. $|\mathbf{V_A}| = |\mathbf{V_B}|$ = speed = υ

If s is the arc length, then Sin θ = s/r
For small θ (θ → 0), sin θ ≅ θ in rads.
Hence θ = s/r

For s = r, θ =s/r = r/r = 1 radian
For s = 2πr, θ =2πr/r = 2π rads ≡ 360°

Notably, since the object continually changes its direction, its *velocity vector* also continually changes i.e., velocity, $\mathbf{V_A}$ at point A has a different direction from $\mathbf{V_B}$ at point B. This velocity is always at a tangent to the circular path and as such, it is the *tangential velocity*.

When the object travels from point A to B, it covers a linear distance, ***s*** (= arc length). The linear speed (υ) of the object = arc length AB/Δt = tangential velocity (***v***) i.e. for small values of ***s***.

$$\mathbf{v} = \frac{d\mathbf{s}}{dt} \qquad \text{……………………………………..(1.16)}$$

Additionally, between point A and B, the object sweeps through an angular displacement θ (measured in *rads*). *The rate of change of θ with time, t, is the angular velocity*, ω i.e.

$$\omega = \frac{d\theta}{dt} \text{ rads}^{-1} \qquad \text{…………………………………….(1.17)}$$

Also since θ = s/r, $\Rightarrow \omega = \frac{d}{dt}\left(\frac{s}{r}\right) = \frac{1}{r}\frac{ds}{dt} = \frac{v}{r}$

Thus: $v = r\omega$ ……..…………...(1.18)

I radian is an angle subtended by an arc length equal to radius of the circle.

A body in circular motion has both *tangential/linear velocity* (***v***) as well as *angular velocity* (**ω**). *The acceleration associated with the angular velocity is the angular acceleration* (**α**),

$$\boldsymbol{\alpha} = \frac{d\omega}{dt} = \frac{1}{r}\frac{d\mathbf{v}}{dt} \qquad \text{…………………………………..(1.19)}$$

The acceleration associated with tangential velocity is the tangential acceleration $\boldsymbol{\alpha}_{tang}$.

$$\boldsymbol{\alpha}_{\mathbf{tang.}} = \frac{d\mathbf{v}}{dt} = r\frac{d\omega}{dt} = r\boldsymbol{\alpha} \qquad \text{………………………….…(1.20)}$$

Due to the changing velocity vector (refer to Fig. 1-17), the body experiences another component of acceleration apart from the tangential acceleration. Since the radius (***r***) and the angular velocity (ω) are constant during the entire motion, the magnitudes of $\boldsymbol{V_B}$ and $\boldsymbol{V_A}$ (= speed) are equal (= υ). Thus the change in velocity from point A to B (refer to Fig. 1-17) is $\boldsymbol{\Delta V = V_B - V_A = V_B + (-V_A)}$ as shown in Fig. 1-18.

Therefore: Acceleration $|\mathbf{a}| = \Delta V/\Delta t = PR/\Delta t$

But $PR = |PQ|\theta = |RQ| = \boldsymbol{v}\,\theta$

$\Rightarrow \quad |\mathbf{a}| = \boldsymbol{v}\theta/t = \boldsymbol{v}\omega$

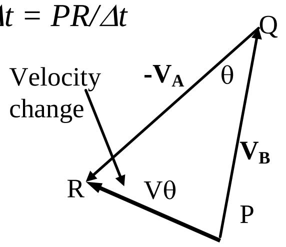


Fig. 1-18

Thus:

$$|\mathbf{a}| = \boldsymbol{v}\omega \qquad \text{..............................(1.21)}$$

Or by substituting for $\boldsymbol{v} = \boldsymbol{r}\omega$

$$\Rightarrow \quad |\mathbf{a}| = a_{cent} = \frac{v^2}{r} = \omega^2 r \qquad \text{..............................(1.22)}$$

This acceleration is directed towards the centre of the circle i.e. it is normal to the velocity vector and it is termed the *normal/radial* or *centripetal acceleration*. Thus, the total acceleration (***a***) for an object in circular motion is a vector sum of $\boldsymbol{\alpha_{tang}}$ *and* $\boldsymbol{a_{cent}}$

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_{tang.} + \mathbf{a}_{cent.} = \frac{d\mathbf{v}}{dt} + \frac{\mathbf{v}^2}{r} = r\boldsymbol{\alpha} + \frac{\mathbf{v}^2}{r} \qquad \text{..............(1.23)}$$

- The force associated with the centripetal acceleration is the *centripetal force*. It *acts towards the centre and keeps the body moving in a circle*. It is given by:

$$F_{cent} = m\mathbf{a}_{cent} = m\omega^2 r = \frac{mv^2}{r} \qquad \text{..............(1.24)}$$

1.6.1 Non -uniform Circular Motion

If the velocity of an object changes both in its magnitude (speed) and direction as the object moves along a circular path, the total acceleration and hence the centripetal force also changes.

For example, for motion in a *vertical circle*, the contribution due to gravitational attraction alters the tension in the string at various points (Fig. 1-19). By applying Newton's 2nd law of motion (***F*** = M***a***, where ***F*** is the net force) we have:

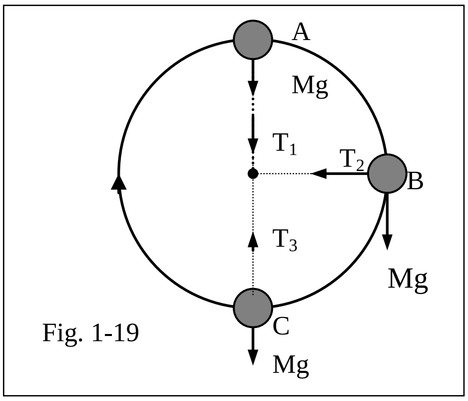


Fig. 1-19

At point A: $F = T_1 + Mg = \frac{Mv^2}{r}$

At point B: $F = T_2 = \frac{Mv^2}{r}$

At point C: $F = T_3 - Mg = \frac{Mv^2}{r}$

Thus, the string is most likely to break at the bottom (point C) since the tension T_3 is greatest.

Practical Examples of Circular Motion

(i) Car on a flat track

Consider a car of mass *M* travelling round a circular track (corner) of radius *r* with a velocity ***v*** (Fig. 1-20a). Whereas the centrifugal force tries to make the car fly away from the track (overturn), the frictional forces often provide the necessary centripetal force to overcome the centrifugal force.

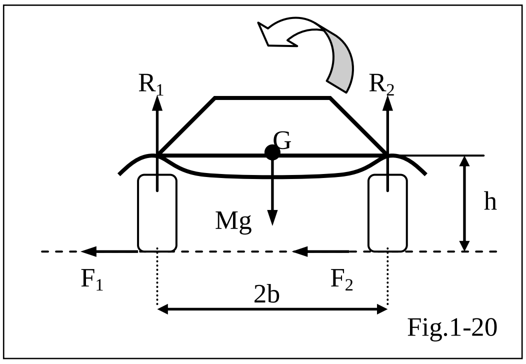


Fig.1-20

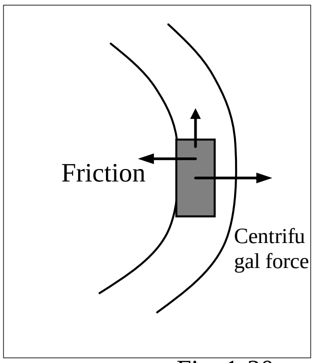


Fig. 1-20a

If the height of the car's center of gravity is *h*, the distance between the wheels is 2*b*, the normal reactions at wheels A and B are R_1 and R_2 respectively, while $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are the corresponding frictional forces (Fig. 1-20), then:

$$R_1 + R_2 = Mg \quad \text{and} \quad F_1 + F_2 = \frac{Mv^2}{r}$$

Taking moments about G, we have $(F_1 + F_2)h = R_2 b - R_1 b$

$$\Rightarrow \qquad R_2 = \tfrac{1}{2}M\left(g + \frac{v^2 h}{rb}\right) \qquad \text{……………….………………..(a)}$$

$$R_1 = \frac{1}{2} M\left(g - \frac{v^2 h}{rb}\right) \qquad \text{..................................(b)}$$

From (b), if $v^2 = \dfrac{brg}{h}$, then R_1 becomes zero and a car turning left will overturn outwards (maximum safe speed is $v^2 < \dfrac{brg}{h}$). To reduce the centrifugal force further, roads are usually banked at corners at some angle.

(ii) The Solar System

- The moon and satellites are held in orbit around the earth by the invisible but real force-the centripetal force provided by the gravitational attraction.

(iii) Looping Loops

- This is another typical case of vertical circular motion (see Tut 1.4, Qn 13)

(iv) Centrifugal Machines

- The cream from milk, honey from honey combs, drying cloths in a spin dryer-all work on the principle that due to centrifugal forces, lighter particles acquire more velocity and hence separate out from the heavier ones.

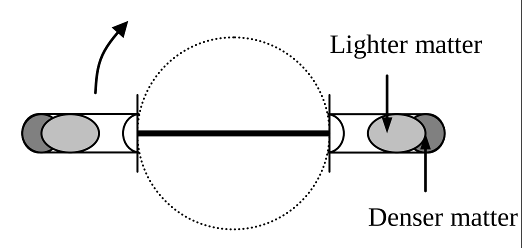


Fig. 1-23. The centrifuge

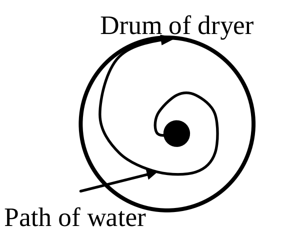


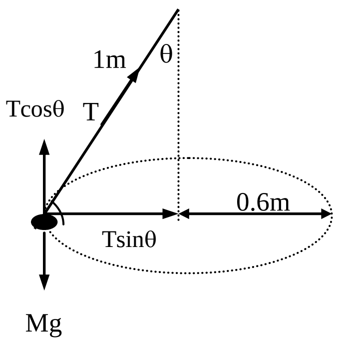
Fig. 1-24

- In a dryer (Fig. 1-24), as the cloths go round, the fibres of the clothing cannot provide sufficient force on most of the water to make it go round. As a result, the water flies off at a tangent to the orbit and escapes through the holes in the drum wall.

Worked Examples

1. A pendulum bob of mass 1.0 kg is attached to a string 1.0m long and made to revolve in a horizontal cycle of radius 0.6m. Find the period of the motion and the tension of the string (assume g = 10 ms^{-2}).

Solution



From the Figure,

$$T \sin\theta = \frac{Mv^2}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots(a)$$

$$T \cos\theta = Mg \quad \ldots\ldots\ldots\ldots\ldots\ldots(b)$$

But $\cos\theta = 0.6$; $\theta = \cos^{-1} 0.6 = 53.1^o$

$$\Rightarrow \quad T = \frac{Mg}{\cos\theta} = 16.67N$$

From (b)

$$\Rightarrow \quad v = \sqrt{\frac{rT\sin\theta}{M}} = \ldots ms^{-1}$$

Thus Period $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = \ldots\ldots\ldots s$

2. A particle moves so that its position vector is given by $\boldsymbol{r} = \cos \omega t\, \boldsymbol{i} + \sin \omega t\, \boldsymbol{j}$ where ω is a constant. Show that
   (a) the velocity, $\boldsymbol{v}$, of the particle is perpendicular to $\boldsymbol{r}$,
   (b) the acceleration, $\boldsymbol{a}$, is directed toward the origin and has magnitude proportional to the distance from the origin
   (c) $\boldsymbol{r} \times \boldsymbol{v}$ = a constant vector.

Solution

(a) $v = \frac{dr}{dt} = -\omega \sin \omega t\, \boldsymbol{i} + \omega \cos \omega t\, \boldsymbol{j}$.

Thus $\boldsymbol{r}.\boldsymbol{v} = [\cos \omega t\, \boldsymbol{i} + \sin \omega t\, \boldsymbol{j}].[-\omega \sin \omega t\, \boldsymbol{i} + \omega \cos \omega t\, \boldsymbol{j}]$
$= (\cos \omega t)(-\omega \sin\omega t) + (\sin\omega t)(\omega \cos\omega t) = 0$

(b) $\frac{d^2r}{dt^2} = \frac{dv}{dt} = -\omega^2 \cos \omega t\, \boldsymbol{i} - \omega^2 \sin \omega t\, \boldsymbol{j} = -\omega^2 [\cos \omega t\, \boldsymbol{i} + \sin \omega t\, \boldsymbol{j}] = -\omega^2 \boldsymbol{r}$

Thus acceleration is opposite the direction of $\boldsymbol{r}$ i.e., it is directed towards the origin. Its magnitude is proportional to $|r|$.

(c) $\boldsymbol{r} \times \boldsymbol{v} = \omega \boldsymbol{k}$, a constant vector

Physically, the motion is that of a particle moving on the circumference of a circle with constant angular speedω. The acceleration, directed towards the centre of the circle is the centripetal acceleration.

TUTORIAL 1.4

1. A ball of mass 0.1Kg is thrown vertically upwards with an initial velocity of $20ms^{-1}$. Calculate the time taken to return to the thrower and the maximum height reached.

2. A ball is dropped from a cliff top and takes 3.0s to reach the bottom below. Calculate (i) the height of the cliff and (ii) the velocity acquired by the ball

3. With what velocity must a ball be thrown upwards to reach a height of 15m?

4. A man stands on the edge of a cliff and throws a stone vertically upwards at 15 ms^{-1}. After what time will the stone hit the ground 20m below?.

5. An object of mass 0.5 kg on the end of a string is whirled round in a horizontal circle of radius 2 m with a constant speed of 10 ms^{-1}.
   (a) Find its angular velocity and the tension in the string.
   (b) If the same object is whirled in a vertical circle of same radius with same speed, what are the maximum and minimum tensions in the string?

6. An object of mass 8.0 kg is whirled round in a vertical circle of radius 2.0 m with a constant speed of 6.0 ms^{-1}. Calculate the maximum and the minimum tensions in the string.

7. A ball tied to a string of length 0.5m swings in a vertical circle under the influence of gravity. If the velocity of the ball is 1.5 ms^{-1} when the string makes an angle of 20^o with the vertical, determine
   (i) the radial component of the acceleration at this instant
   (ii) the total acceleration (magnitude) and its direction at this instant..

8. A particle moves in circle of radius 20 cm. If its tangential speed is 40 cm/s, find (a) its angular speed and its angular acceleration (b) its normal acceleration.

9. A particle moving on a circle of radius R has a constant angular acceleration α. If the particle starts from rest, show that after time t
   (a) its angular velocity is $\omega = \alpha t$, (b) the arc length covered is $s = \frac{1}{2} R\alpha t^2$.

10. Show that the acceleration of a body moving in a circular path of radius r with uniform speed v is v^2/r and hence draw a diagram to show the direction and variation of this acceleration with distance r.

11. Water in a bucket is whirled round in a vertical circle. Under what condition will the water stay in the bucket.

12. A blade of a circular saw of radius 3m is initially rotating at 7,000 revolutions per minute. The motor is then switched off and the blade coasts to a stop in 8.0s. Determine the average angular acceleration and the initial centripetal acceleration.

13. Calculate the speed at which a plane must be flying when looping-the-loop of radius 0.80 km so that the pilot feels no force from either his harness or his seat.

14. A hump-back bridge has a radius of curvature of 40 m. Calculate the maximum speed at which a car could travel across the bridge without leaving the road at the top of the hump.

15. A ball rolls off the edge of a horizontal table 4m high. If it strikes the floor at a point 5m horizontally away from the edge of the table, what was its speed at the instant it left the table.

16. A stone is thrown horizontally at 20 ms^{-1} from the top of a cliff, which is 300 m high. Draw accurately the following graphs: $\boldsymbol{a_x}$ *-t*, $\boldsymbol{v_x}$ *-t*, $\boldsymbol{x}$ *-t;* $\boldsymbol{a_y}$ *-t*, $\boldsymbol{v_y}$ *-t*, $\boldsymbol{y}$ *-t*.

17. Water emerges horizontally from a hose-pipe with velocity 4.0 ms^{-1}. The pipe is pointed at P on a vertical surface 2 m away from the pipe (Fig 1.). If the water strikes at S, calculate the distance PS.

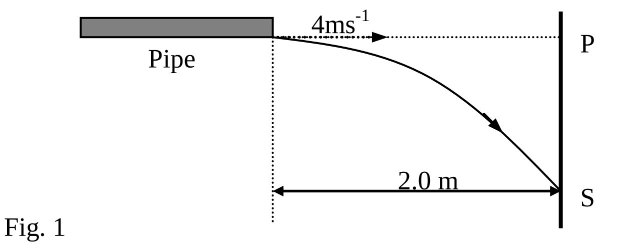


Fig. 1

18. A projectile is hauled at a horizontal distance R from the edge of a cliff of height h in such a way as to land a horizontal distance x from the bottom of the cliff (Fig. 2). If x is to be large as possible, what should be the magnitudes of θ and the initial velocity (*U*), assuming that *U* can be varied continually from zero to some finite maximum value. Only one collision with the ground is allowed.

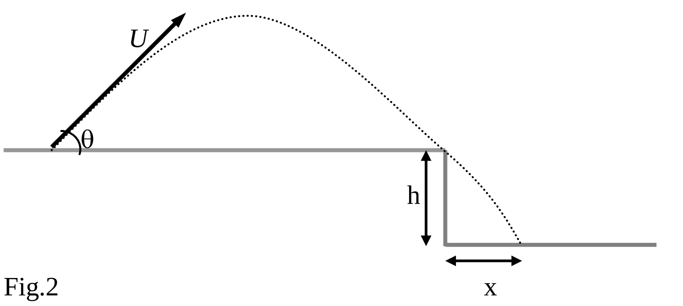


Fig.2

Suggested Further Reading

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7. HALLIDAY AND RESNICK., **Physics -Part one**.

Module Two: DYNAMICS

Fundamental Physical Forces
Force and Newton's Laws of Motion
Linear Momentum
Motion in a resisting medium
Work, Energy and Power
Collisions and Energy

USEFUL RELATIONSHIPS

Dynamics

Force,	$F = ma$
Newton's 2nd Law of Motion,	$F = \frac{dL}{dt}$
Linear Momentum,	$L = mv$
Frictional Force,	$F_k = \mu_k N\ ;\ F_s \le \mu_s N$
Work, Energy conversion	$W = F.dr = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta K.E = \Delta P.E$
Kinetic Energy,	$E_K = \frac{1}{2}mv^2$
Potential Energy,	$E_P = mgh$
Conservative Forces,	$\oint_c F.dr = 0$
Non-conservative Forces,	$\oint_c F.dr \ne 0$
Power,	$P = \frac{dW}{dt} = F.v$
Impulse,	$I = \int Fdt = mv - mu$

DYNAMICS

What causes a body to accelerate?
The answer to such questions takes us into the subject of **dynamics**, the study of the relationship between motion and the force that causes it. These relationships are given by Newton's *three* laws of motion (also called *the laws of Dynamics*).

Assessment Objectives

- *At the end of this module, candidates should be able to*

(ii) *State each of Newton's laws of motion*
(iii) *Demonstrate an understanding that mass is the property of a body which resists change in motion*
(iv) *Use the relationship F = ma to solve problems involving linear motion*
(v) *Define force as rate of change of momentum*
(vi) *State the principle of conservation of momentum*
(vii) *Explain conservation of momentum in terms of Newton's third law*

2.1 Fundamental Physical Forces

- Although many types of forces exist in nature such as gravity, friction, tension, electric, magnetic etc., all these type of forces (macroscopic and microscopic) can be classified into four types of fundamental forces namely Gravitational, Electromagnetic (E-M), Strong and Weak Nuclear Forces. These fundamental forces are responsible for all properties and changes affecting matter and come into play both in the vastness of the cosmos and in the infinite smallness of atomic structures.

(i) ***Gravitational forces****:*

- It’s the weakest of the fundamental forces and it is the mutual attraction between masses. It affects large objects e.g., planets, stars, galaxies etc. and it is very weak on the level of atoms.

- Gravity is finely tuned. If gravity decreased, the stars would increase in size and the pressure of gravity in their interiors would not drive the temperature high enough for nuclear fusion reactions to get underway: The sun would not shine, life sustaining oxygen would escape from the earth's atmosphere and surface water would evaporate. In either case, there would be no life on earth. What would be the consequences of a stronger force of gravity?

(ii) ***Electromagnetic forces****:*

- It is about 10^{14} times the force of gravity. It is the force of attraction or repulsion between electric charges e.g. protons and electrons and allows molecules to form. Lighting is one evidence of its power. Every force in our every-day macroscopic environment is electric. The contact force between rigid bodies, elastic forces, pressure forces, friction forces etc. are nothing but electric forces between charged particles in the atoms of one body and those in the atoms of another body.

- If E-M forces were significantly weaker, atoms would not combine to form molecules. Conversely, if this force were much stronger, electrons would be trapped on the nucleus of an atom and as such, there could be no chemical reactions between atoms-meaning no life. On a cosmic scale, a slight difference in E-M force would affect the sun and the light reaching the earth thereby making photosynthesis in plants impossible. It could also rob water of its unique properties.

(iii)***The `Strong Nuclear Forces'***:

- It is the strongest of the fundamental forces and acts mainly within the nuclei of atoms i.e., it binds the nucleus by gluing the protons and neutrons. Because of this bonding, various elements can form (e.g., helium, oxygen, lead etc). Notably, if this force were only 2% weaker, only hydrogen would exist and if it were slightly stronger, only heavier elements could be found (i.e., no water or food since hydrogen is an essential ingredient of both).

(iv)***The `weak forces'***:

- These forces manifests itself mostly in special reactions among elementary particles such as in radioactive decay and the efficient thermonuclear activity of the sun. Further, the weak force plays a role in supernova explosions, which they give as the mechanism for producing and distributing most elements of which you and I are composed. It is 10^6 times weaker than the strong nuclear force.

The weak forces are just weak enough so that the hydrogen in the sun burns at a slow and steady rate. What would happen if this force were much stronger?

2.1.1 Fundamental Forces and the Universe

The Universe contains at least 50 billion galaxies each with billions of stars (like our sun). The sun and all the individual stars you 'see' belong to the *Milky Way galaxy*. All the galaxies are in motion-receeding away from each other indicating that the universe is expanding. An expanding universe would have profound implications about our past and perhaps our personal future too. Something must have started the process-a force powerful enough to overcome the immense gravity of the entire universe. Scientists believe that the source of such dynamic energy could be traced to a very small, dense beginning (a singularity) i.e., at some point, the universe was once close to a singular state of infinitely small size and infinite density. No one has told us what was there before and what was outside the universe-A problem of the beginning.

The rate of expansion of the universe seems very finely tuned. If the universe had expanded one million millionth part faster, then all the material in the universe would have dispersed by now. If it had been a million millionth part slower, then gravitational forces would have caused the universe to collapse within the first thousand million years or so of its existence. However, explaining this initial singularity-where and when it all begun, still remains the most intractable problem of modern cosmology. [*Nature just got it right:- the precise fine-tuning of the fundamental forces has made possible the existence and operation of the universe and may explain either the origin or the early development of the universe*].

2.2 Force and Newton's Laws of Motion

Force

- A *force (**F**) is a pull or a push exerted on a physical object by an external agent* or it is that quantity which causes change in motion, form or size of an object. ***F*** is a *vector* with units, the *Newton* (N). *A Newton is the force that gives a mass of 1 Kg an acceleration of 1ms^{-2}* i.e., 1N = 1Kgms^{-2}

2.2.1 Newton's Laws of Motion

Law I

This law describes the natural state of motion of a body on which no external forces are acting. It states as follows

- *A body tends to remain at rest or in uniform motion in a straight line (i.e., with constant velocity) unless acted upon by a resultant force*. The tendency of a body to continue in its initial state (a state of rest of uniform velocity) is its *inertia*. Accordingly, the first law is often called the *law of inertia*.

The 2nd and 3rd laws deal with the behaviour of bodies under the influence of external forces.

Law II

- When a net force acts on a body thereby causing motion (acceleration), *the magnitude of the acceleration produced is proportional to the net external force (F) and inversely proportional to the mass (m) of the object.*

  NB: For a constant mass (m), the greater the force, the greater the acceleration. Thus

$$F \propto a \qquad \text{……………………………(a)}$$

  For a bigger mass (m), a greater force is needed to cause the same magnitude of acceleration (a) i.e.,

$$F \propto m \qquad \text{……………………………(b)}$$

  Combining (a) and (b) we have

$$F_{ext} = ma \qquad \text{……………………………(2.1)}$$

Law III

- *Action and reaction are always equal and opposite*. The repulsion of rockets, motion of vehicles, walking etc. relies on the concept of action and reaction.

Note*:** Newton's laws of motion apply only to objects moving with speed (v***) << ***c*** (the speed of light) and are valid only in an inertial frame of reference (the earth is approximately good example of such a frame).

The motion of particles whose speed (***v***) ~ ***c*** is best described by *quantum mechanics* and the *special theory of relativity*. Why?

2.2.2 Linear Momentum & Newton's Laws of Motion

- The *linear momentum (**L**) of an object a property of the object and it is the product of its mass (m) and its velocity (v)* i.e.,

$$\boldsymbol{L} = m\boldsymbol{v} \quad (\boldsymbol{L} \text{ is a vector}). \qquad \text{…..……………………..…....(2.2)}$$

Newton's laws can be defined very neatly in terms of momentum as follows:

Law I

- Law I describes the natural state of a body on which no external forces are acting. Consequently, the body is at rest and its momentum is zero and shall remain zero as long as $\boldsymbol{F_{ext}} = \boldsymbol{0}$. Thus:
  - *In the absence of external forces, the momentum of a particle remains constant (is conserved).*

Law II

In presence of external forces, an object aquires an acceleration (a) according to Eqn (2.1). If we assume that the mass (*m*) of an object is independent of time (i.e., remains constant always -Newtonian Mechanics), then Eqn. (2.1) can be written as

$$F_{ext} = ma = \mathrm{m}\frac{d\mathrm{v}}{d\mathrm{t}} = \frac{d(mv)}{dt} = \frac{dL}{dt}$$

$$\Rightarrow \qquad F = \frac{dL}{dt} \qquad \text{…….…………………..……..………(2.3)}$$

- *Thus, the rate of change of momentum of a body is directly proportional to the net **external force** that produces it and takes place in the direction of the force.*

Law III

- "*Whenever two bodies exert forces on each other, the resulting changes of momentum are of equal magnitudes and of opposite directions*".

2.2.3 Newton's Laws and Conservation of Linear Momentum

Any of the Newton's laws of motion cam be used to show that linear momentum is always conserved as shown below.

Law I

- *From section 2.2.4 above, we see that in the absence of external forces, the momentum of a particle at rest remains zero (constant). Thus, L is conserved.*

Law II

- For an object at rest (L = 0) or moving with constant velocity v (L = mv), then, *if no external forces act on an object, its momentum remains constant* i.e.,

From $F = \frac{dL}{dt} = \frac{d(mv)}{dt}$

$\Rightarrow$ When $\boldsymbol{F} = 0$, then $\frac{dL}{dt} = 0$

$\Rightarrow$ $L = m\boldsymbol{v}$ = constant.

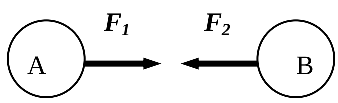
Thus: Newton's 1st law of motion is only a special case of the 2nd law.

This conservational law is termed *The Principle of conservation of linear momentum.* This principle is useful in solving problems involving collisions between bodies.

Law III

Consider a system of two interacting particles (force of interaction between them can be either coulombic or gravitational). If $\boldsymbol{F_1}$ is the force exerted by particle *A* on particle *B* (i.e., action) while $\boldsymbol{F_2}$ is the force exerted by particle B on particle A (i.e., reaction), then by Newton's third law

$$\boldsymbol{F_1} = -\boldsymbol{F_2}$$



But from 2nd Law, $F_1 = \frac{dL_B}{dt} = M_B \frac{dV_B}{dt}$ and $F_2 = \frac{dL_A}{dt} = M_A \frac{dV_A}{dt}$

where M_B, V_B and L_B are the mass, velocity and momentum of particle B while M_A, V_A and L_A are the parameters of particle A.

But it follows from Newton's 3rd law that

$\boldsymbol{F_1 = -F_2}$

or $\frac{dL_B}{dt} + \frac{dL_A}{dt} = 0$ or $\int \frac{d}{dt}(L_A + L_B) = 0$

$\Rightarrow$ $\boldsymbol{L_A + L_B}$ = zero (constant).

Thus, from Newton's 2nd and 3rd laws, it can be said that *the total linear momentum of a system of particles free from the action of external forces and subjected only to their mutual interaction remains constant, no matter how complicated the forces are*.

2.3 Mass and Weight

Mass is the quantity of matter in a body (measured in Kg) i.e., it is an intrinsic property of a body, measuring the inertial resistance with which the body opposes change in its motion. The mass of a body is the same regardless of its position in the universe.

Weight (***W***) is an extrinsic property measuring the pull of gravity on a body. It depends on the gravitational environment of the body in the universe. *Weight* is a force directed towards the centre/origin of the earth or terrestrial body. For object of mass m above the earth's surface, $F = W = Mg$.

2.4 Motion in a Resisting Medium

Forces that act to oppose the motion of an object in some medium e.g., air or water are termed *resisting, damping* or *dissipative forces*. The corresponding medium is thus a *resisting or damping medium*. If $\boldsymbol{R}$ is the resistance, then the motion of a particle of mass m in an otherwise uniform (gravitational field) is given by

$$m\frac{d^2\mathbf{r}}{dt^2} = mg\mathbf{k} - \mathbf{R} \qquad \text{..(2.4)}$$

where **k** is a unit vector indicating direction of mg

One of the most important resistive forces is **friction**. *Friction is the force that always acts to oppose relative motion between surfaces.* For example, in a case involving relative motion between two surfaces e.g., a block being pulled on a metal surface (Fig. 2-1), the force of friction (F_r) between the two surfaces opposes the applied external force (F)

The frictional force (F_r) is always proportional to the normal force (N) acting on the surfaces but is independent of the area of contact and the relative speed i.e., $\boldsymbol{F_f \propto N}$. *i.e.,*

$$\boldsymbol{F_r = \mu N} \qquad \text{..(2.5a)}$$

where μ is the coefficients of friction

For a stationary object, the frictional force has a maximum value and no motion occurs until the external force (F) exceeds the maximum frictional force (F_f). This frictional force is called *Static Friction* and is given by

$$\boldsymbol{F_s = \mu_s N}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.5b)$$

Once the object is in motion, the frictional force reduces to a value $\boldsymbol{F_k\ (=\mu_k N)}$ called the *Kinetic friction.*

$$\boldsymbol{F_k = \mu_k N}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.5c)$$

μ_k and μ_s are the coefficients of kinetic and static friction respectively ($\mu_s > \mu_k$ always). Both μ_s and μ_k are dependent on the type of surface.

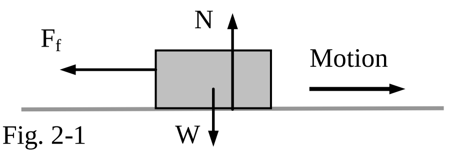


Fig. 2-1

NB: *for objects moving at low speed through liquids and gases, the frictional forces are often approximately proportional to the speed of the object through the fluid.*

2.4.1 Application of friction

(a) *Walking*:- This is made possible due to friction between the feet/shoes and the ground otherwise people would just slip and fall.

(b) *Match sticks*:- Fire results from friction between the stick and rough surface.

(c) *Locomotives*:- The tyres of a car usually push backwards against the road. The force of friction opposes this and pushes forward on the tyres, making the car move forward.

Questions for Discussion

1. A body of large mass is suspended by means of a string A from the ceiling and another similar string B is attached to the bottom of the body. Which string would break when string B is pulled (a) sharply, and (b) steadily? Why?.

2. Comment on these two statements: (a) Frictional forces always oppose motion. (b) the motion of nearly all motor car is caused by friction.

3. Discuss the principles of physics involved in (a) putting the shot (b) throwing the discuss (c) throwing the javelin (d) the pole vault

4. Why does a car with soft springing lean backwards when accelerating?

5. What is the resultant force on an astronaut travelling in a space ship in a circular orbit round the earth?

6. Two men want to break a cord. They first pull against each other. Then they tie one end to a wall, and pull together. Which procedure is better?

TUTORIAL 2.1

1. A man carrying a 20Kg sack on his shoulders rides in an elevator. Find the force that the bag exerts on his shoulders if the elevator accelerates upwards at $2ms^{-2}$.

2. A pilot in a fast moving jet aircraft loops a loop of radius 1.5Km. If the speed of the aircraft is 200m/s when it passes through the bottom of the loop, what is the apparent weight that the pilot feels?

3. A 60-Kg woman stands on a scale on the floor of an elevator. What is the reading on the scale when
   (i) the elevator is at rest
   (ii) the elevator is accelerating upward at $1.8ms^{-2}$
   (iii) the elevator is moving upwards with constant velocity
   (iv) the elevator cable is cut.

4. An aircraft of mass 20×10^{3}Kg lands on an aircraft-carrier deck with a horizontal velocity of $90ms^{-1}$. If it is brought to rest in a distance of 100m, calculate the (average) retarding force acting on the plane.

5. A 5.0 Kg mass moves on a smooth horizontal surface under the action of a horizontal force given by $F = 80 + 10t^{2}$. Determine
   (a) the velocity of the mass at t = 3.0s if it was at the origin at t = 0s.
   (b) the displacement of the mass when t = 2.0s.

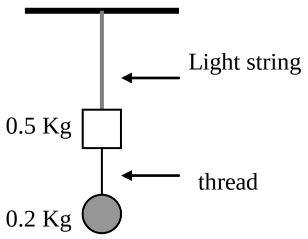
6. A metal ball of mass 0.5 Kg drops from the top of a vertical cliff of height 90m. If it hits the beach below and penetrates to a depth of 6.0 cm. Calculate
   (i) the velocity acquired by the ball just as it hits the sand
   (ii) the (average) retarding force of the sand ($g = 10ms^{-2}$).

7. Due to a force field, a particle of mass 5 units moves along a space curve whose position vector is given as a function of time t by $r = \left(2t^{3} + t\right)i + \left(3t^{4} - t^{2} + 8\right) j + 12t^{2}k$. Find
   (i) the velocity and the momentum of the particle
   (ii) the acceleration and hence the force field at any time t.

8. A block of mass m slides down a frictionless incline making an angle θ with an elevator floor. Find its acceleration relative to the incline when
   (i) the elevator descends at constant speed
   (ii) the elevator ascends at constant speed
   (iii) the elevator descends with an acceleration of $\boldsymbol{a}$ ms^{-2}.
   (iv) the elevator cable is cut
   (v) What is the force exerted on the block by the incline

9. An 80g stone is released at the top of a vertical cliff. After falling for 3s, it reaches the floor of the cliff and penetrates 9cm into the ground. Determine
   (i) the height of the cliff
   (ii) the average force resisting penetration of the ground by the stone.

10. The system in the figure is in static equilibrium. If the thread is burned out, calculate the magnitude and direction of the acceleration of the 0.5 Kg mass.



11. A particle of mass 2 units moves in a force field depending on time t given by $F = 24t^2\, i + (36t - 16)\, j - 12t\, k$. Assuming that at t = 0, the particle is located at $r_o = 3i - j + 4k$ and has velocity $V_0 = 6i + 15j - 8k$. Determine
    (a) the velocity at any time t (b) the position of the particle at any time t

12. While thinking of Isaac Newton, a man standing on bridge overlooking a highway inadvertently drops an apple over the railway just as the front end of a truck passes directly below the railing. If the vehicle is moving at 55 km/h and is 12m long, how far above the truck must the railing be if the apple just misses hitting the rear end of the truck.

13. A 2 Kg block is dropped from a height of 0.4m to a spring whose force constant is 1960 Nm^{-1}. Find the maximum distance the spring will be compressed.

2.5 Work, Energy and Power

Assessment Objectives

- *Candidates should be able to*

(i) Express work as a product of Force and distance
(ii) Derive and use $\boldsymbol{E_K = 1/2mv^2}$, $\boldsymbol{E_P = Mgh}$, $\boldsymbol{W = F.r}$
(iii) Use the relationship between force and Potential energy in a uniform field

(iv) Appreciate the importance of energy losses in practical devices and use the concept of efficiency
(viii) Identify Power as ***P = Fv*** *and relate it to work using appropriate examples*
(i) Give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation to simple examples
(ix) Appreciate that reserves of non-renewable sources such as oil and gas are finite, and show an awareness of the importance of alternative renewable sources such as hydro-electric, geothermal, tidal, solar, wind and wave

Energy and work are closely related and we shall first introduce the concept of work

Work

- *Work is done when energy is transferred from one system to another* and it involves a force moving its point of application along its line of action i.e.,

$$\mathbf{W\ (J) = (Force)(displacement\ )}\ = F.r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\textbf{(2.6)}$$

Energy

In everyday language, energy means a capacity for rigorous activities and hard work. Likewise, in the language of Physics, *Energy is the capacity to do work*.

- Energy is "stored" or latent work, which can be converted into actual work under suitable conditions. Machines get their energy from fuels (oils, gas, etc.) when they are burnt. Man gets his energy from food.

Forms of energy

(i) *Chemical energy*:- Found in foods, oils, charcoal, biogas etc. and is due to the K.E and P.E of the electrons within atoms.
(ii) *Mechanical energy* :- It’s the energy of motion
(iii) *Wave energy*:- It's due to vibration of objects or particles
(iv) *Thermal energy*:- It is associated with the heat produced from the sun, heaters, burning fuels, etc.
(v) *Electrical energy*:- Associated with the electric charge. It is produced by generators from hydroelectric power stations, tides, Geothermal stations, nuclear fission etc.
(vi) *Solar energy*:-It's the energy associated with electromagnetic waves.

- Remarkably, when a body loses one form of energy, this energy never disappears but it is merely translated into other forms of energy e.g., vehicles burn fuels (chemical energy) to produce both thermal (heat) and mechanical energy.

2.5.1 Mechanical energy

- *It is the energy of motion -whether that energy is in action or stored*. It exists in two forms namely;

(i) ***Kinetic Energy (K.E)***

- *It is the energy possessed by a body in motion and it represents the capacity of the body to do work by virtue of its speed.*

- For example, if a force $\boldsymbol{F}$ acts on an object of mass m such that the mass accelerates uniformly from rest ($\boldsymbol{u}$ = 0 ms^{-2}) to a velocity, $\boldsymbol{v}$ ms^{-2} over a distance, $\boldsymbol{s}$, (Fig. 2-2), then:

$$\text{Work } (W) = \boldsymbol{F}.\boldsymbol{s}$$

But $\boldsymbol{F} = m\boldsymbol{a}$ and $\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\mathbf{a}t^2$. Since $\boldsymbol{u} = 0$ and $\mathbf{a} = \boldsymbol{v}/t$,

$$\Rightarrow \quad \mathbf{s} = \tfrac{1}{2}(\mathbf{v}/t)t^2 = (\mathbf{v}t)/2$$

$$\Rightarrow \quad W = ma\,(\mathbf{v}t/2) = \tfrac{1}{2}m\mathbf{v}^2$$

Thus work done on object = K.E. gained by the object i.e.,

$$\text{K.E.} = \tfrac{1}{2}mv^2 \qquad \text{……………...………...…….……..(2.7)}$$

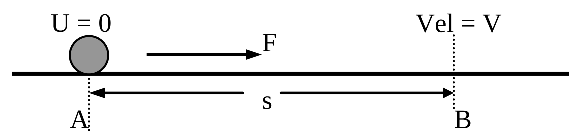


Fig. 2-2

If the path between the two point A and B is nonlinear (Fig. 2-3) then the total work done between points A and B can be obtained using a line integral. If $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ are the position vectors of points A and B while $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are the velocities at points A and B respectively, then

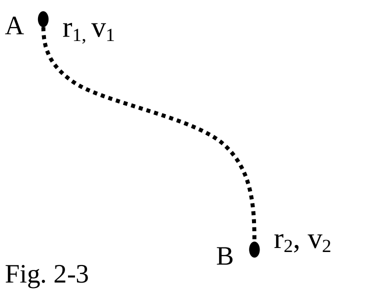


Fig. 2-3

$$W = \int_A^B F.dr = \int_{r_r}^{r_2} F.dr$$

But $F = m\frac{dV}{dt}$

$$\Rightarrow \quad W = \int_{r_1}^{r_2} m\frac{dV}{dt}dr = m\int_{V_1}^{V_2} VdV$$
$$= \tfrac{1}{2}m\left[V^2\right]_{V_1}^{V_2}$$

Thus $$W = \tfrac{1}{2}m\left(v_2^2 - v_1^2\right) = \Delta K.E \quad \text{……………..………(2.8)}$$

Generally, work done by a force = K.E gained or lost by the object and Eqn. (2.8) is called the ***work-energy equation.***

(ii) ***Potential Energy (P.E.)***

This is the energy possessed by a body by virtue of its configuration (position) in a force field (*e.g*., gravitational field, electrostatic field etc).

For example, if an object of mass (m) is lifted to a height (h) from the ground (Fig. 2.4) then:

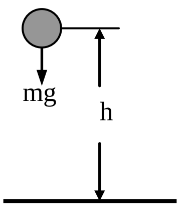


Fig. 2-4

Work (W) done on the mass by external agent $= \boldsymbol{F}.h$

But $\boldsymbol{F}$ = wt. of object = mg (Force needed to lift the mass without causing acceleration)

$\Rightarrow W = mgh$

$\Rightarrow$ work done on the object = gain in P.E by object.

Generally, $$P.E. = mgh \quad \text{………..…………………..……………..…..(2.9)}$$

Note: *The unit of work and energy is the Joule* (J)*, which is the work done by a force of 1* N *during a displacement of 1m. Work and Energy are scalars.*

2.5.2 Conservative Forces and Conservation of Energy

Conservational laws play an important role in the world of matter. Such laws assert that some quantity is conserved i.e., remains constant even when matter suffers drastic changes involving motions, collisions and reactions. One familiar example is the conservation of mass which, asserts that the mass of a given particle remains constant

Other conservational laws involving mechanical quantities are the conservation of Energy (which is one of the most fundamental laws of nature) and Momentum. These laws are helpful in making predictions about some aspects of motion of particles when it is impossible to obtain the full details of the motion from Newton's 2nd law e.g., in cases where the forces are not known exactly.

A force field is said to be conservative if (i) the total work done in moving round a closed non-intersecting loop is zero and is path independent and the total energy of the system is always conserved. i.e.,

$$\oint_C F.dr = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.10)$$

A gravitational field is a good example of a conservative force fold. If an object of mass *m* falls in a gravitational field from point A (height *h* above the ground) to point B (height *x* above the ground) as shown in Fig. 2-5, then;

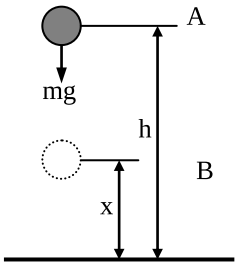


Fig. 2-5

P.E at point A = *mgh*; ***K.E*** at point A = 0
$\Rightarrow$ Total Energy at point A = *mgh*

P.E at point B = *mgx*; ***K.E*** at B = $\frac{1}{2} mv^2$
Using $v^2 = u^2 + 2as$, where $u = 0$
and $s = (h-x) \Rightarrow v^2 = 2g(h-x)$
$\therefore$ Total energy at B = $mgx + mg(h-x)$
$= mgh$

$\Rightarrow$ Total E at A = Total E at B

Thus, at any point in a gravitational field, the total mechanical energy of a falling object is conserved and is equal to the original energy [*Principle of Conservation of Mechanical Energy*]. The general law states: *Although energy may transform from one form to another, total energy remains constant.*

Thus in a gravitational force field, the total energy (U) is a function of position only, U(x), say. Thus F is related to the total energy u(x) by the equation

$$W = \int_c F.dr = MgxP = U$$

$$\Rightarrow \qquad F = -\frac{dU}{dx}$$

In a non-conservative force field, the total work done in moving round a closed non-intersecting loop is not zero i.e.,

$$\oint_C F.dr \neq 0 .$$

In this case, some energy is lost against dissipative (frictional) forces.

Power

Generally, Power is the time rate at which a source of energy does work i.e.,

$$P = \frac{dW}{dt} \qquad \text{………………..……………(2.11)}$$

Also, since $W = F.dr$

$$\Rightarrow \quad P = \frac{dW}{dt} = F \cdot \frac{dr}{dt} = F \cdot v \qquad \text{……………..……………..…(2.12)}$$

Or since F = rate of change of momentum

$$\Rightarrow \quad P = \frac{dW}{dt} = F \cdot \frac{dr}{dt} = mv\frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad P = V\frac{dL}{dt}$$

Energy sources are limited not only by the total amount of energy available, but also by the rate at which it can be delivered. The rate at which energy is converted from one form to another (or work done by the source) is its power with units Joules per second or *watts (W)*. In engineering practice, power is often measured in horsepower (hp) where 1 hp = 746 W. This is roughly the rate at which a (very strong) horse can do work.

Efficiency

Usually, if s certain amount of energy is supplied to an object, the source of energy does not necessarily lose an equivalent amount, but it loses more than the minimum energy required since energy is unavoidably dissipated as heat. Thus, there are always losses in energy transfer. Even in electrical power transmission, the cables are heated by the current and the amount of energy received at the end of the transmission line is less than the amount delivered at the beginning.

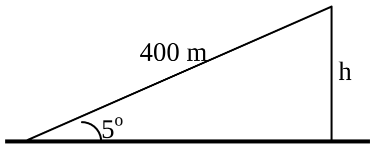
The efficiency (ε) of any energy transfer system is the ratio of energy received to the energy delivered by the source or it is the ratio of its power output to power input. Efficiency is usually quoted in percentage e.g., electrical transmission lines have a very high efficiency of about 99% while internal combustion engines have a efficiency of about 28% due to heat losses. The variable output power of the internal combustion engine is one of the reasons for having gears in a car. The gear ratios are chosen so that the engine is running at a speed, not too far from the speed at which it can deliver peak power. This improves on the engine efficiency at converting fuel energy into mechanical work.

Notably, the way you drive also determines the efficiency of your car engine-high speeds and frequent acceleration is uneconomical.

Worked examples

1. A man of 80 Kg climbs up a slope 400m long inclined at 5^o to the horizontal. Calculate the minimum work done by the man.

Solution



Work = F(dist) = Wt of man x h
But h = 400 sin 5^o
Thus W = 800 x 400 sin 5^o
=2.79×10^4J

2. *Calculate the work done by a builder in lifting a stone of mass 15 Kg through a height of 2.0m (g =10ms^{-2})*

Solution: W = **F**.**r**, but F = wt of stone = mg = 150 N
Thus $W = mg(r)$ = 150(2) = 300 Nm = 300 J

3. A car of mass 1 x 10^3 kg traveling at 72 km/h on a horizontal road is brought to rest a distance of 40 m by the action of brakes and frictional forces. Find the average braking force and the time taken to stop the car.

Solution

(i) Work done by braking force = Change in K.E i.e $\boldsymbol{Fs} = \frac{1}{2} \boldsymbol{mu^2}$

$\Rightarrow$ $\boldsymbol{F} \times 40\text{m} = \frac{1}{2} \times 1 \times 10^3 \text{ kg} \times (20 \text{ ms}^{-1})^2$ since $\boldsymbol{v} = 0$

$\Rightarrow$ $\boldsymbol{F} = 5 \times 10^3$ N

Or $\boldsymbol{F = ma}$: By using $\boldsymbol{v^2 = u^2 + 2as}$, where $\boldsymbol{v}$ = 0ms^{-1}, $\boldsymbol{u}$ = 20 ms^{-1}, $\boldsymbol{s}$ = 40m, we get $\boldsymbol{a}$ = - 5 ms^{-2} Thus $\boldsymbol{F = ma} = 5 \times 10^3$ N

(ii) Using $\boldsymbol{v = u + at}$, $\Rightarrow$ $\boldsymbol{t}$ = 4.0 s.

4. Show that a force field ***F*** defined by $F = (y^2z^3 - 6xz^2)i + 2xyz^3 j + (3xy^2z^2 - 6x^2z)k$ is a conservative force field.

Solution: ***F*** is conservative if $\oint F \cdot dr = 0$

$$\Rightarrow \quad V = -\int_{r_o}^{r} F \cdot dr = \int_{(x_o,y_o,z_o)}^{(x,y,z)} \left(y^2z^3 - 6xz^2\right)dx + 2xyz^3dy + \left(3xy^2z^2 - 6x^2z\right)dz$$

$$= 6x^2z^2 - 2xy^2z^3 + C \qquad \text{where } C = 2x_o y_o^2 z_o^3 - 6x_o^2 z_o^2$$

5. The most powerful sports car you can buy can reach 100 Km/h in 4 seconds. What is the maximum power output, assuming the mass of the car to be 1 tonne.

   Solution:

   Average power = $\dfrac{Gain\ in\ K.E}{Time\ taken} = \dfrac{\frac{1}{2}mv^2}{t}$ = 100Kw.

Questions for Discussion

1. Explain why it is easy to become physically tired when pushing a car even when you fail to move it (and therefore fail to do any work).
2. Mechanical ***k.e***. and ***p.e***. are conserved only when conservative forces act. How then can total energy always be conserved?
3. Explain why a heavy lorry may have the same maximum speed as a family car on a flat, straight road but a far lower max speed when going up a steep.

TUTORIAL 2.2

1. A block of mass 3.0 Kg is pulled 5.0 cm up a smooth plane by a force of 25N. If the plane is inclined at 30^o to the horizontal, find the velocity of the block when it reaches the top of the plane.
2. Find the work done in moving an object along a vector $r = 3i + 2j - 5k$ if the applied force is $F = 2i - j - k$.
3. A truck of mass 150 Kg is released from rest at point A and it moves along a frictionless truck *ABCD* (see Fig. 1). Determine
   (i) the maximum kinetic energy acquired by the truck
   (ii) the maximum velocity of the truck
   (iii) the velocity of the truck at point C (assume g = 10 ms^{-2}).
   (iv) what happens when the truck reaches point D

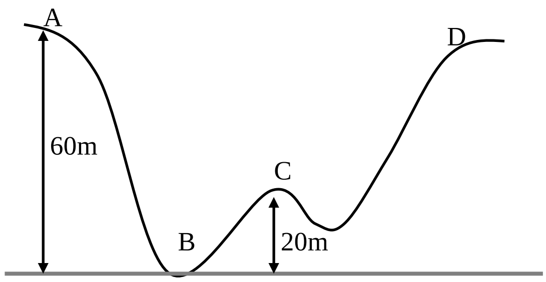


Fig. 1

4. A resultant force of 30 N is applied for a 4.0 s to a body of mass 10 kg which was originally at rest. Calculate
   (i) the distance traveled,
   (ii) the work done on the body, and hence its final ***k.e***., and
   (iii) the final velocity of the body.

5. (a) A 0.2Kg ball drops from a height of 10m and rebounds to a height of 7.0 m. Neglecting air resistance, calculate the energy loss on impact with the floor.
   (b) A ball of mass 0.1Kg is thrown vertically upwards with an initial velocity of $20ms^{-1}$. Calculate the Kinetic and potential energy of the ball half way up. (assume g = $10ms^{-2}$).
6. Fig. 2a shows the variation of speed *v* with time *t* for a short journey traveled by a car of mass 800 kg. Calculate
   (a) The acceleration of the car during the first 20 s of the journey and the resultant force that acted on the car during this interval
   (b) The distance traveled by the car during the first 80s of the journey
   (c) During the interval PQ, no resultant force acted on the car. State one reason why power is required to maintain zero resultant force
   (d) During the interval RT, the car was involved in a minor traffic accident. State, with a reason, the likely nature of the accident

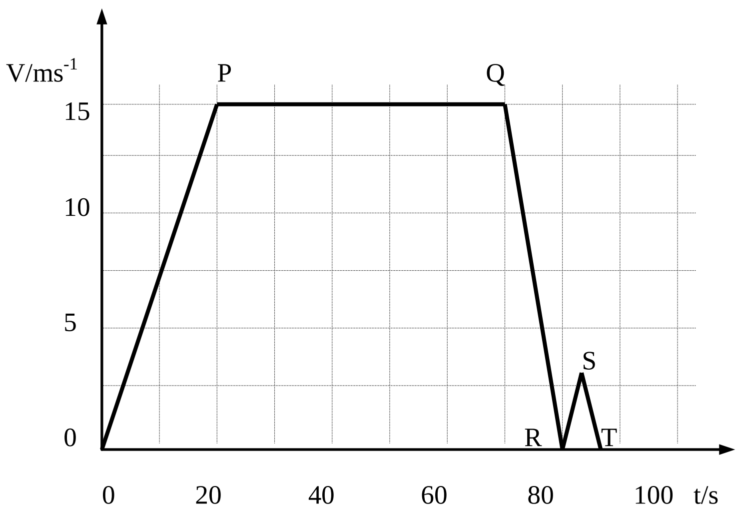


Fig. 2a

7. An object of mass 1.5 Kg is thrown vertically upwards with a velocity of 25 ms^{-1}. If 10% of its initial energy is dissipated against air resistance on its upward flight, calculate (a) Its maximum kinetic energy (b) the height to which it will rise.

8. One end of a light inelastic string of length *l* is attached to a small body of mass *m*, and the other end is fixed. The body is released from a position when the string is taut and horizontal. Determine the instantaneous values of the following parameters when the string is vertical.
   (i) The kinetic energy and velocity of the body
   (ii) The tensional force exerted by the string.

9. A particle of constant mass m moves in space under the influence of a force field F. Assuming that at times t_1 and t_2, the velocities of the particle are V_1 and V_2 respectively, show that the work done by the particle is equal to the change in the kinetic energy of the particle i.e., $\int_{t_1}^{t_2} F.dr = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$.

10. 20 x 10^3 kg of water moving at 2.2 ms^{-1} is incident on a water wheel each second. Calculate the maximum power output from the mill assuming 40% efficiency.

11. Prove that if F is a force acting on a particle and V is the velocity of the particle, then the power applied to the particle is given by P = F.V.

12. Show that the force field ***F*** defined by $F = \left(y^2z^3 - 6xz^2\right)i + 2xyz^3 j$ + $\left(3xy^2z^2 - 6x^2z\right)k$ is a conservative force field.

13. Find the kinetic energy of a particle of mass 20 units moving with velocity $V = 3i - 5j + 4k$.

14. A car of mass 20×10^3 Kg moves up an incline at a steady speed of 15 ms^{-1} against a frictional force of 0.6 KN. The incline is such tat it rises 1.0m for every 10m along the incline. Calculate the output power of the car engine.

15. A stone of mass 2.0 Kg falls vertically from rest through a distance of 25 m under the influence of gravity. Sketch graphs of
   (a) the work done by the gravitational force, and
   (b) the power (P) of the force both as a function of time. When is P -ve?

16. An upward force of 10 kN accelerates a lift from rest at 2.0 ms^{-2} for 5.0s. Calculate
   (i) the average power required during the whole time interval,
   (ii) the instantaneous power after 1.0 s and 4.0 s.

17. A body of mass 4.0 kg falls from rest. Assuming that all the potential energy of the body is converted into Kinetic energy,
   (a) find the speed of the body after it has fallen 5.0m.

(b) In practice, a falling body does not accelerate indefinitely. Describe and explain the motion of a falling body near the surface of the Earth

18. Fig. 2 represents a frictionless big dipper truck. A small truck is released at R and allowed to move to the right. Determine
    (a) The speed of the truck at S (b) The speed of the truck at T
    (c) With what speed should the truck be released from Q in order to reach S ?

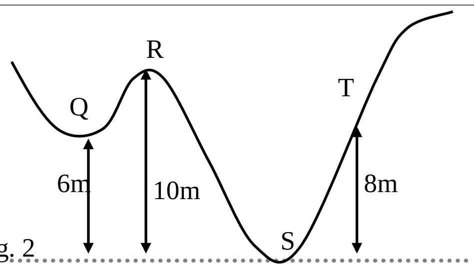


Fig. 2

2.6 Collisions and Energy

Assessment Objectives

At the end of this section, you should be able to

(i) Apply the principle of conservation of momentum in simple cases including elastic interactions between bodies in two dimensions and inelastic interactions in one dimension

2.6.1 Linear Momentum & its Conservation

From Newton's 2nd law (Eqn. 2.2, pp 38), we saw that *if no external force acts on an object then, its momentum* is conserved i.e.,

$$F = \frac{dL}{dt} = 0 \quad \Rightarrow \quad \boldsymbol{L} \text{ is constant}$$

This principle is useful in solving problems involving collisions between bodies. For example, suppose a bullet of mass m is fired with velocity $\boldsymbol{v}$ from a gun of mass M. If the gun recoils with velocity $\boldsymbol{V}$ then:

Momentum of the bullet in forward direction = $m\boldsymbol{v}$
Momentum of the gun in backwards direction = $-M\boldsymbol{V}$
$\Rightarrow$ Total momentum = $m\boldsymbol{v} + M\boldsymbol{V} = 0$

Hence final momentum of gun and bullet = Total initial momentum of gun and bullet i.e., the linear momentum is conserved.

Impulse

- *The product of the force and time is called the impulse (I)* of the force i.e.

$$I = Ft = Mv - Mu = \text{change in momentum} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.14)$$

Alternatively, the impulse can be given as the time integral of the force (F) i.e.,

$$I = \int_{t_1}^{t_2} F dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.15)$$

2.6.2 Collisions

In physics, collision does not necessary mean physical contact between bodies. When two bodies approach each other, a force comes into play between them for a finite time bringing about a change in their velocities, momenta and energy according to the conservational laws. A collision is said to have occurred.

In all collisions, the Momentum as well as the Total Energy are conserved. However, ***k.e*** might not be conserved since it may be converted into other forms of energy like sound, heat or work done during plastic deformation. There are two main types of collisions: *elastic* and *inelastic.*

(a) Elastic collisions (Scattering)

In this case *both the kinetic energy and momentum are conserved*. Thus, from fig. 2-6 it follows that:

$$M_A U_A + M_B U_B = M_A V_A + M_B V_B \quad \text{(conservation of momentum)} \quad \ldots\ldots\ldots(a)$$

$$\tfrac{1}{2} M_A U_A^2 + \tfrac{1}{2} M_B U_B^2 = \tfrac{1}{2} M_A V_A^2 + \tfrac{1}{2} M_B V_B^2 \quad \text{(cons. of } k.e\text{)} \quad \ldots\ldots\ldots(b)$$

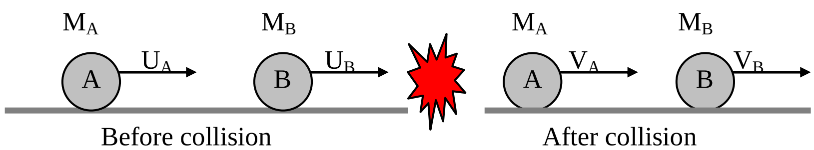


Fig. 2-6

(b) Inelastic collisions

In this case, *Kinetic energy is not conserved but momentum is conserved*. Thus:

$$M_A U_A + M_B U_B = M_A V_A + M_B V_B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(c)$$

If the colliding objects stick together, the collision is *totally inelastic i.e*,

$$M_AU_A + M_BU_B = (M_A + M_B)V \quad \text{..............................(d)}$$

A collision is termed scattering if the nature of particles does not change after the collision e.g. the deflection of a comet as it passes near the solar system or the deflection of an α-particle by an atomic nucleus.

Special cases

(c) Elastic collisions between equal masses

(i) *Collisions in a straight line.*

For elastic collision in a straight line between objects A and B, (with B initially stationary, $U_B = 0$) and $M_A = M_B = M$, then, equations (a) and (b) become

$$MU_A = MV_A + MV_B \quad \text{..(e)}$$

and

$$\tfrac{1}{2}M_AU_A^2 = \tfrac{1}{2}MV_A{}^2 + \tfrac{1}{2}MV_B{}^2 \quad \text{...................................(f)}$$

from (e) $\Rightarrow U_A = V_A + V_B$

from (f) $\Rightarrow U_A^2 = V_A{}^2 + V_B{}^2$

Solving gives $U_A = V_B$ and $V_A = 0$.

Thus the two objects simply exchange velocities i.e. object A comes to rest while B moves off with the original velocity of A. This is a situation of maximum energy transfer between two colliding bodies and is mostly applicable in nuclear reactions where neutrons are stopped by protons.

(ii) Oblique collisions of equal masses

If mass A collides obliquely with B, which is at rest ($U_B = 0$), then the total momentum of any of the masses will be the sum of the respective momentum components in the vertical and horizontal directions respectively (Fig. 2.7).

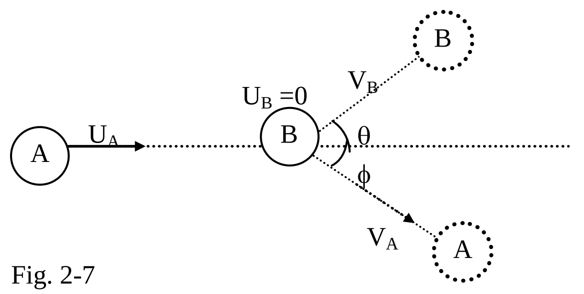


Fig. 2-7

In the diagram, θ is the *angle of scattering* i.e, the angle between the initial and final directions of the incident particles after collision while ϕ is the *recoil angle* (angle between the direction of target (or recoil) particle after collision and initial direction of the incident particle).

Thus, Conservation of momentum gives

$MU_A = MV_A \cos\phi + MV_B \cos\theta$ (along the +ve *x* direction) ………..(g)
$0 = -MV_A \sin\phi + MV_B \sin\theta$ (along the +ve *y* direction) ……..…(h)

Conservation of K.E. gives

$$\tfrac{1}{2} M_A U_A^2 = \tfrac{1}{2} MV_A{}^2 + \tfrac{1}{2} MV_B{}^2 \quad \text{………………………………(i)}$$

Eqns. (g) - (i) can then be solved for parameters of interest.

Note: If $M_A = M_B = M$ and M_B was at rest ($U_B = 0$), then both particles move in directions perpendicular to each other after an elastic collision i.e., $\theta + \phi = 90^o$.

(iii) Recoil
In cases where part of a composite body suddenly flies apart e.g. a bullet fired from a gun, repulsion of rockets, jet engines etc, the remaining part (the gun in this case) must undergo motion in the opposite direction (recoil) in order to conserve the momentum according to Newton's 3rd law of motion. If M_b and V_b are the mass and velocity of the bullet while M_g is the mass of the gun, then, the gun will recoil with velocity V_g given by;

$$M_b V_b = M_g V_g \quad \text{Or} \quad V_g = -\frac{M_b V_b}{M_g}$$

$$\Rightarrow \quad V_g << V_b \quad \text{since } M_b << M_g$$

Worked examples

1. A car travelling at 90Km/h slams into a tree and is stopped in 40 ms. If the car has a mass of 800 Kg, calculate the average force acting on the car during the collision.

   Solution $v = 90Km/h = 25m/s$; *From* **Ft = mv-mu**

   $$\Rightarrow \quad 0.04\,\mathbf{F} = 800(25) \quad \therefore\ \mathbf{F} = 5\times10^5\ N$$

2. A marble of mass 25g moving horizontally at 10 m/s collides head-on with a 40g marble moving with a velocity of 6 m/s in the opposite direction. If the collision is elastic, find their respective velocities after collision.

   Solution.
   From conservation of momentum;
   $M_A\boldsymbol{U_A} + M_B\boldsymbol{U_B} = M_A\boldsymbol{V_A} + M_B\boldsymbol{V_B}$.
   $\Rightarrow$ 0.025Kg (10) + 0.04Kg (-6) = 0.025Kg (V_A) + 0.04Kg (V_B) ……………..(j)
   From conservation of K.E;
   $\tfrac{1}{2} M_A U_A^2 + \tfrac{1}{2} M_B U_B^2 = \tfrac{1}{2} M_A V_A^2 + \tfrac{1}{2} M_B V_B^2$.

$\Rightarrow$ ½(0.025Kg)(100) + ½(0.04Kg)(-36) = ½ (0.025Kg)V_A + ½ (0.04Kg)V_B ….(k)

Solving Eqns (j) and (k) gives $\boldsymbol{V_A}$ = -9.69m/s and $\boldsymbol{V_B}$ = 6.31m/s

3. A stationary golf ball is hit with a club, which exerts an average force of 80N over a time of 0.025s. Calculate
   (i) the change in momentum
   (ii) the velocity acquired by the ball if it has a mass of 0.020 Kg.

Solution

(i) Change in momentum = Impulse = **Ft** *= (80)(0.025) = 2Ns 0r Kgms^{-1}.*

(ii) Change in momentum = M**v** - M**u** $\Rightarrow$ *2 = (0.02)***v** *- 0 or* **v** *= 100ms^{-1}.*

Questions for discussion

1. Show that the translational kinetic energy and the linear momentum $\boldsymbol{L}$ of a particle of mass *m* are related by $E_K = L^2/2m$.

2. A baby is strapped in a pram which is resting on a horizontal surface. If the baby starts kicking, describe the motion of the pram with (a) the brake off, and (b) the brake on.

3. Explain in terms of physical principles why it is sensible for a car passenger to be wearing a seat belt in a head-on collision. Should the belt be tight or loose?

TUTORIAL 2.3

1. A 2-Kg object moving with a velocity of 8 ms^{-1} collides with a 4.0-Kg object moving with velocity of 5.0 ms^{-1} along the same line. If the two objects join on impact, calculate their common velocity and the loss in the kinetic energy when they are initially moving
   (a) in the same direction
   (b) in the opposite direction

2. A 2 Kg object moving with a velocity of 6.0 ms^{-1} collides with a stationary object of mass 1.0 Kg. Assuming that the collision is perfectly elastic, determine the velocity of each object after the collision.

3. A mini-bus of mass 1500Kg travelling at 72 km/h collides with a stationary car of mass 900Kg. If the impact takes two seconds before the two move together at a constant velocity for 20 seconds, calculate
   (i) the common velocity and the distance moved after impact
   (ii) the force of impulse during collision
   (iii) K.E before and after collision

4. A bullet of mass m and velocity $\boldsymbol{v}$ passes through the bob of a simple pendulum of mass *m*, and emerges with velocity $\frac{1}{2}\boldsymbol{v}$. If the length of the pendulum string is *l*, calculate the minimum value of $\boldsymbol{v}$ such that the bob will describe a complete circle.

5. A particle of mass 4.0g is moving in a straight line at 1.0kms^{-1}. It experiences a collision of duration 3.0μs in which the direction of motion is deviated through 30^{o}, but the speed is unchanged. Calculate
   (a) the change in momentum and
   (b) the magnitude of the force responsible.

6. The weight of a rocket is 80KN at the instant it takes off and the products of combustion are ejected at a velocity of 0.60 kms^{-1} relative to the rocket. Calculate the minimum rate at which the rocket must be consuming fuel in order to take off vertically.

7. A golf club collides elastically with a golf ball and gives it an initial velocity of 60 ms^{-1}. If the ball is in contact with the club for 15 ms, and the mass of the ball is 2.0×10^{-2} kg. Calculate the magnitudes of
   (a) the impulses given to the ball and club respectively
   (c) the initial ***k.e***. of the ball
   (d) the average force exerted on the ball by the club, and
   (e) the work done on the ball. Note the relation between this answer and your answer to (c).

8. A box moving at 0.4 ms^{-1} on a horizontal frictionless track accumulates sand which falls into it vertically at the rate of 5.0 gs^{-1}.
   (i) What horizontal force is required to keep the box moving with constant velocity
   (ii) Describe quantitatively how sand would fall into the box so that no horizontal force is required to maintain motion

9. A man on a sheet of smooth ice sets himself in motion by throwing successfully his two boots, each of mass m, in the same direction with velocity v relative to himself. Calculate the man's final velocity if his mass without his boots is M.

10. Show that in an elastic collision between particles of equal masses, one of which is initially at rest, the recoiling particles always move off so that the angle between the directions of their velocities is $\pi/2$ rad. Can you reconcile your answer with the result of a head-on collision?

11. A steel ball is held above a horizontal table and released so that it falls on to the table and rebounds several times. If the collision is elastic, sketch a graph that represents the variation of the ball's acceleration *a* with time *t*.

Suggested Further Reading

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12. NELKON M. and PARKER P., **Advanced Level Physics**, *7th Ed*.
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14. HALLIDAY AND RESNICK., **Physics -Part one**.

Module Three: STATICS

Centre of Mass
Equilibrium of Parallel Forces
Couple and Torque

USEFUL RELATIONSHIPS

Static Equilibrium

Centre of mass,	$\bar{x} = \frac{\int xdm}{\int dm}$
Translational Equilibrium,	$\Sigma F_x = \Sigma F_y = \Sigma F_z$
Rotational Equilibrium,	$\Sigma \Im_x = \Sigma \Im_y = \Sigma \Im_z$
Moment of Force,	$M\, r \times F$
Torque (due to F and -F),	$= 2\, r \times F$
Work done by Torque,	$W = \Im \theta$

Statics is the oldest branch of physics and it studies the conditions for the equilibrium of a body. Engineers and architects concerned with the design of bridges and other structures need to know under what conditions a body will remain at rest, even when forces act on it. Egyptians, Greeks and Romans had a good grasp of the basic principles of statics, as evident from their construction of elegant arches for doorways and bridges.

We shall assume in the first part of this chapter that "rigid" structural members do not deform when subjected to action of forces. The most common forces met in equilibrium situations include
(i) weight
(ii) Tension (or compression forces)
(iii) Friction:- force between surfaces
(iv) Normal (reaction) forces: - Acts on an object when it is trying to push into another object. They act perpendicular to the surface.

Assessment Objectives

- *Candidates should be able to*

(k) Use a vector triangle to represent forces in equilibrium
(l) Understand a couple as a pair of forces tending to produce rotation only
(m) Define and use the moment of force and the Torque of a couple
(n) Show that when there is no resultant force and no resultant torque, a system is in equilibrium
(o) Apply the principle of moments to simple systems of parallel forces acting in a plane
(p) Draw free body diagrams for particles and for extended bodies

3.1 Centre of Mass (C.M.)

- *The centre of mass of an object may be defined as the point at which an applied force produces acceleration but no rotation.*

- In Fig. 3-1 below, particles of masses m_1, m_2, m_3,….m_n, together form the object of total mass M. If x_1, x_2, x_3, ……x_n, are the respective x-coordinates of the particles relative to the axes O_x, O_y, then generally, the coordinates $\bar{x}$ and $\bar{y}$ of the C.M. can be given by

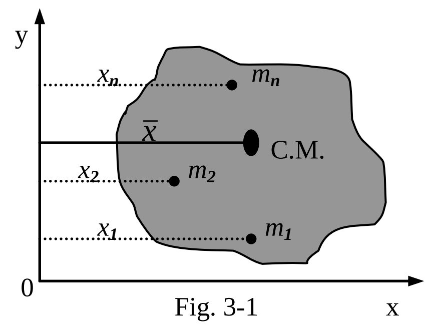


Fig. 3-1

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 ....+ m_n x_n}{m_1 + m_2 ....+ m_n} = \left\{ \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \right\} = \frac{\sum_{i=1}^{n} m_i x_i}{M} \qquad ....(3.1)$$

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 ....+ m_n y_n}{m_1 + m_2 ....+ m_n} = \left\{ \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} \right\} = \frac{\sum_{i=1}^{n} m_i y_i}{M} \qquad ....(3.2)$$

And the position vector of the C.M. is given by

$$\bar{R} = \bar{x} + \bar{y} = \frac{\sum_{i=1}^{n} m_i r_i}{M} \qquad ....(3.3)$$

- In the presence of an applied force ***F***, the velocities and acceleration of the C.M. can be given by

$$\bar{v} = \frac{d\bar{R}}{dt} = \frac{d}{dt}\left(\frac{\sum_{i=1}^{n} m_i r_i}{M} \right) = \frac{\sum_{i=1}^{n} m_i v_i}{M} \qquad ....(a)$$

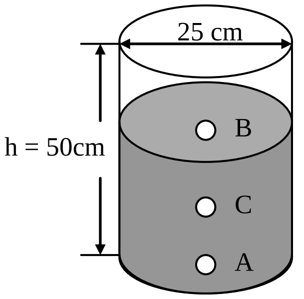
$$\bar{a} = \frac{d^2 \bar{R}}{dt^2} = \frac{\sum_{i=1}^{n} m_i a_i}{M} \qquad ....(b)$$

- *The Centre of gravity (C.G.) of a body on the other hand is the point where the resultant force of attraction or weight of the body acts*. A body is in stability of equilibrium when its potential energy is minimum or the C.G. is lowest.

Worked example

1. A cylindrical can is made of a material of specific mass 10g cm^{-2}. and has no lid. The diameter of the can is 25 cm and its height is 50 cm. Find the position of the centre of mass when the can is half full of water.

Solution



Area of base = πr^2
=490.87 cm2

Mass of base = $(\pi r^2)(10g)$ = 4908.74 g (acts at pt A, the centre of the base)

Mass of curved surface of cylinder = $2\pi rh(10g)$ = 39,269.91g
and acts at pt B, half way along the axes.

Mass of water = $\pi r^2 (h/2)$ = 12271.85g and acts at C, the mid-point of AB.

$\Rightarrow$ Resultant mass = 56,450.50g

Assuming that the C.M is $\bar{x}$ distance from the base (point A) then by taking moments at A we have

$$56{,}450.50\,(\bar{x}) = 39{,}269.91\,(AB) + 12271.85\,(AC)$$

$$\Rightarrow \quad \bar{x} = 20 \text{ cm.}$$

Thus the C.M is 20 cm from the base.

2. Two bodies of masses 2Kg and 10Kg have position vectors (3***i*** + 2***j*** - ***k***) and (***i*** - ***j*** + 3***k***) respectively. Find the position vectors and the distance of the centre of mass from the origin.

Solution

Position vector of the C.M. is $\bar{R} = \dfrac{m_1 r_1 + m_2 r_2}{M_1 + M_2} = \dfrac{16i - 6j + 28k}{12}$

Distance of C.M. from origin is $|\bar{R}| = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{7}{3}\right)^2} = 2.73\,Units$

3.2 Equilibrium of Parallel Forces

- Consider a rigid body of weight (W) carrying a weight (W_1) and being supported by two springs of tensions $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ respectively as shown in Fig. 3-1. For the body to remain in equilibrium, *the total upward force (*$\boldsymbol{F_1}$ and $\boldsymbol{F_2}$*) should balance the total downward force* (W and W_1).

- Additionally, forces $\boldsymbol{W_1}$ and $\boldsymbol{F_2}$ act to turn the body in anti-clockwise direction about point O while $\boldsymbol{F_1}$ acts to turn the body in a clockwise direction. This turning effect of a force is the *moment*.

- *The moment (M) of a force about a point is the product of the force (**F**) and the perpendicular distance from the point to the line of action of the force* i.e. from Fig. 3-2.

$$\boldsymbol{M} = \boldsymbol{F}(r \sin\theta) = r(\boldsymbol{F}\sin\theta) = \boldsymbol{F} \times \boldsymbol{r} \qquad \ldots\ldots(3.3)$$

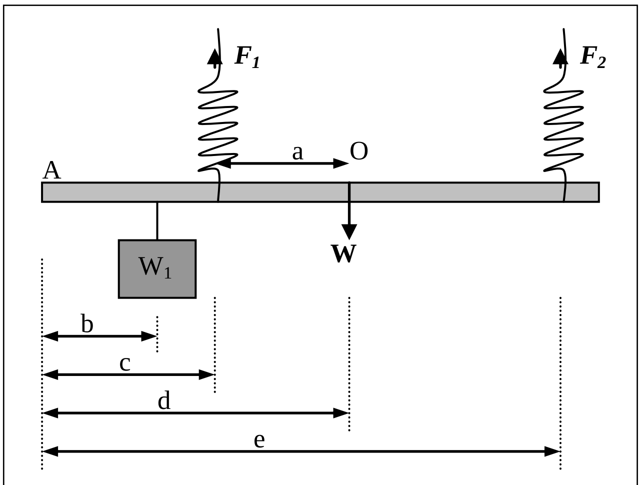


Fig. 3-1. Parallel forces

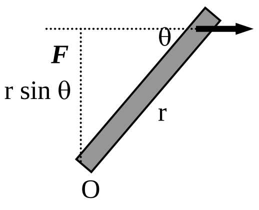


Fig. 3-2.

Thus, for static equilibrium of a rigid body, the sum of clockwise moments about any point = the sum of anti-clockwise moment about the same point (called *principle of moments*) i.e., for moments about point A

$(W_1 \times b) + (W \times d) = (F_1 \times c) + (F_2 \times e)$
(clockwise moments) (anti-clockwise moments)

Generally, the necessary and sufficient conditions for static equilibrium are

(i) The sum of external forces on the rigid body must be zero i.e., $\sum F_x = \sum F_y = \sum F_z$

(i) The sum of external torques on the body must be zero (i.e. sum of clockwise moments about any point = the sum of anti-clockwise moment about that point; $\sum \Im_x = \sum \Im_y = \sum \Im_z$).

3.2 Couple and Torque

A couple is a pair of equal and parallel forces acting on a body in opposite direction e.g. forces applied by a driver on a steering wheel when going round a bend (Fig. 3-3) or when opening or closing a tap (Fig. 3-4).
A couple produces a turning effect called a *torque* ($\Im$) or *the moment of a couple*. For example, in the case of a wheel (Fig. 3-3), we have

Moment (M) of one force = $\mathbf{F} \times \mathbf{r} = \mathbf{F}\,(r \sin \theta) = Fr$ since $\theta = 90^o$.
Total moment due to couple = Torque ($\Im$) = $2\,(\mathbf{F} \times \mathbf{r}) = 2\mathbf{Fr}$

Thus:

$$\Im = 2\mathbf{Fr} = \mathbf{Fd} \qquad \text{……………………………………(3.4)}$$

where $d = 2r$, diameter of the wheel or perpendicular distance between forces

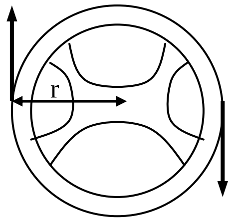


Fig. 3-3. Steering wheel

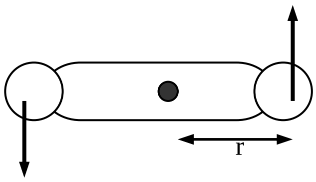


Fig. 3-4. Opening a tap

3.2.1 Work done by a couple

If a force $\boldsymbol{F}$ rotates a wheel through an angle θ rads and the torque due to the couple remains constant, then the work (W) done by the couple can be given by:

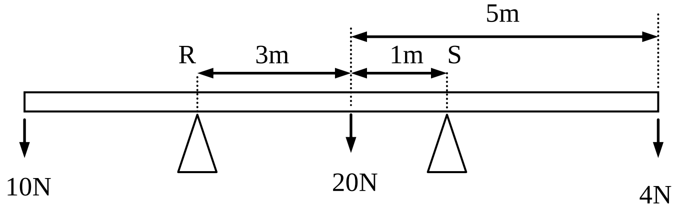
Work (W) = sum of work done by both forces

$$= \boldsymbol{Fr}\,\boldsymbol{\theta} + \boldsymbol{Fr}\,\boldsymbol{\theta} = 2\boldsymbol{Fr}\,\boldsymbol{\theta}.$$ But $2\boldsymbol{Fr} = \Im$, Thus:

$$W = \Im\theta \qquad \text{……………………..……..…………………..…....(3.5)}$$

Worked Example

1. A 10m-length beam is supported at points R and S and carries weights as shown in the figure below. If the beam is in equilibrium, find the magnitude of the reaction at R and S.



Solution

For equilibrium in a vertical direction

$\Rightarrow$ R + S = 10N + 20N + 4N = 34N ……..……………..……………..(a)

Taking moments about point S we have

$\Rightarrow$ (10 × 6) + (20 × 1) = (R × 4) + (4 × 4) ……….………..….(b)

From equations (b) $\Rightarrow$ R = 16N

and from (a) $\Rightarrow$ S = 34 – 16 = 18N

Questions for discussion

1. Was a loaded washing line more likely to break if it was initially tightly stretched between two points or if it sagged quite a lot?

2. The centre of gravity of a body is usually considered to coincide with its centre of mass. Why is this justified? Describe the conditions under which it is not.

TUTORIAL 3.1

1. The top of a ladder of mass 10 Kg and length 10m leans on a smooth wall making an angle of 30^o with the wall while the bottom rests on a rough floor. If the coefficient of static friction between the ladder and the floor is $U_s = 0.5$, determine how high a man of 80 Kg can climb the ladder before it slips. [NB. *When the ladder is about to slip, the frictional force has its maximum value*].

2. The bottom of a ladder of mass M Kg and length L m leans on a frictionless wall while the bottom rests on a rough floor. If the coefficient of static friction between the ladder and the floor is $U_s = 0.4$, what is the maximum angle that the ladder can make with the wall without slipping?

3. A constant couple whose one force is 20N is applied to a wheel of radius 0.04m. If the wheel makes 5 revolutions, find the work done by the couple.

4. A traffic lamp of 25Kg hangs in the middle of a wire stretched between two posts. If the two halves of the wire sag downward at an angle of 20^o as shown in Fig. 1., find the tension in the wire.

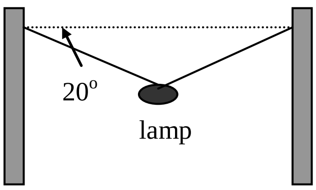


Fig.1

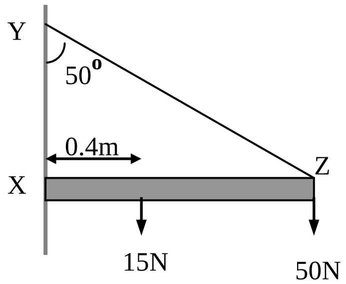


Fig. 2

5. Figure 2 shows a non-uniform rod XZ of weight 15N and length 1.5m held in position by a length of string YZ. A weight of 50N is hung at Z. Calculate the force exerted by the string and the push of the hinge on the rod.

6. A locomotive of 90,000Kg is one third of the way across a bridge 90m long. The bridge consists of a uniform girder of 900, 000Kg which rests on two piers. Find the load on each pier.

7. A particle is acted upon by the forces $\boldsymbol{F}_1 = 5\,\boldsymbol{i} -10\,\boldsymbol{j} +15\,\boldsymbol{k}$, $\boldsymbol{F}_2 = 10\boldsymbol{i} + 25\boldsymbol{j} -20\boldsymbol{k}$ and $\boldsymbol{F}_3 = 15\boldsymbol{i} - 20\boldsymbol{j} + 10\boldsymbol{k}$. Find the force needed to keep it in equilibrium.

8. The top of a ladder of weight W Kg and length L m leans on a smooth wall while the bottom makes an angle α with the ground. Show that a man of weight M Kg will be able to climb the ladder without having it slip if the coefficient of friction between the ladder and the ground is at least $\left(\frac{M+\frac{1}{2}W}{M+W}\right)\cot\alpha$.

3.3 Deformation of Solids

The mechanical properties of a material relate to its response when it is loaded or deformed, i.e., when subjected to stress or strain. To select a material for a particular job, one therefore needs to know its behaviour when subjected to various types of external forces.

Material's mechanical properties of paramount importance to an engineer are the *tensile strength, compression strength, hardness, fatigue* and *creep* and the behaviour of a material are determined under these tests. Tensile, compression and hardness tests are closely related since they measure the same basic physical property -the resistance of materials to gross plastic deformation.

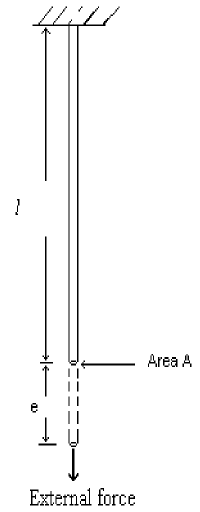
Assessment Objectives

- *Candidates should be able to*

(i) *Describe the behaviour of springs in terms of load, extension, Hook's law and spring constants*
(ii) *Define and use the terms stress, strain and the Young's modulus*
(iii) *Distinguish between elastic and plastic deformation of a material*
(iv) *Deduce the strain energy in a deformed material from the area under load-extension graph*
(v) *Demonstrate knowledge of force-extension graphs for typical ductile, brittle and polymeric materials*

3.4 Tensile Strength

The elastic behaviour is the common measure of mechanical properties and is used to determine the allowable stress in many engineering designs. It involves stretching a material by an external force (Fig. 3-4).



The extension (e) produced in a material depends on

(i) nature of material.
(ii) the stretching force (e ∝ F).
(iii) cross-sectional area (e ∝ 1/A).
(iv) the original length (*l*).

Combining the above factors, it follows that

$$e \propto \frac{Fl}{A}.$$

Fig. 3-4. Stretching a wire

Hooke's law states that the extension, e, produced in a material is directly proportional to the stretching force; **F** provided the elastic limit is not exceeded.

3.4.1 Elastic properties

The *Engineering stress* (σ) and the *Engineering strain* (ε) are defined by

$$\sigma = \left(\frac{\text{Force}}{\text{Original cross sectional area}}\right) = \frac{F}{A_o} \quad \left(Nm^{-2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots..(3.7)$$

$$\varepsilon = \left(\frac{\text{Deformation}}{\text{Original } \textit{specimen} \text{ length}}\right) = \frac{e}{L_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.8)$$

Note. Deformation (Change in length) can be elongation, compression etc.

Hook's law can also be stated in terms of stress and strain as follows. From the expression; $e \propto \frac{Fl}{A}$, it follows that $\frac{e}{l} \propto \frac{F}{A_o}$ *Or* $\frac{e}{l} = E\frac{F}{A_o}$

$$\Rightarrow \quad \boldsymbol{\sigma = \varepsilon E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.9)$$

where *E* is a constant (The Young's modulus of the material)

Thus: *The tensile strain in a material is directly proportional to the tensile stress during elastic deformation [Hook's law]*. A practical application of Hook's law is in spring balances.

The three successive stages in response by a material when stressed are: elastic strain, plastic strain and fracture. Typical stress-strain (or Force-extension) diagrams are shown in Fig. 3-5 for various materials. Regions on Fig. 3-5 are classified as follows:

(a) Elastic limit (OE):

Maximum stress (σ_e) that a material can endure without plastic deformation or it is the region in which the material recovers when the load/stress is removed. In this region the stress $\propto$ strain i.e. Hook's law ($\sigma = \varepsilon E$) is obeyed.

NB. If the specimen is unloaded at point A', the line A'O' is followed. On reapplication of the load, the line O'A' is retraced. In general, the specimen exhibits elastic behaviour over the region O'A' and thus plastic deformation appears to increase the range of elastic behaviour in the deformed specimen. This is called *work hardening*.

(b) Yield strength:

Minimum stress (σ_y) at which the material starts to deform (yields) plastically without a noticeable increase in load (stress) i.e. at point Y.

Plasticity is the property of a material to undergo some degree of permanent deformation without rupture i.e. material is plastic between points Y and E.

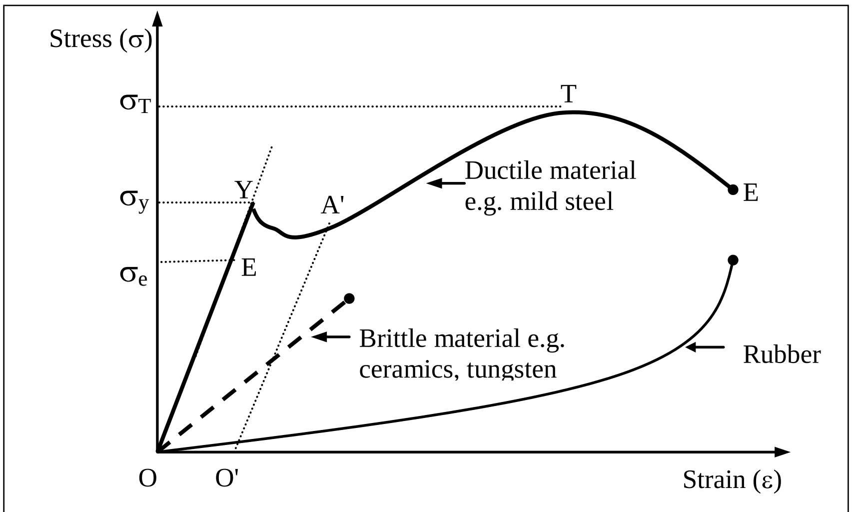


Fig. 3-5 Stress-strain curve for ductile, brittle and polymeric materials

(c) Tensile (ultimate) strength:

The strength (σ_T) corresponding to the maximum load reached before rupture. At this point (D), a tensile specimen becomes unstable in that any section, which extends, and hence contracts in cross-sectional area, becomes the weak link in which all further deformation is concentrated, a process called necking.

(d) Rupture:

Rupture or failure occurs when a material can no longer withstand the applied stress (i.e. point E). The rupture (breaking) strength is the stress (σ_f) required to break a material.

Stiffness

It is the resistance of a material to elastic deformation or deflection and is commonly expressed in terms of the modulus of elasticity (*E*) i.e.,

$$E = \frac{Stress(\sigma)}{Strain(\varepsilon)} = \frac{FL_o}{eA_o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.10)$$

where: *F* = load at Elastic limit and *e* = extension at elastic limit.

3.4.2 True stress and true strain

In actual case, as specimen elongates due to an applied load, its cross-sectional area decreases. Thus:

- *True stress*: The ratio of the applied load to the instantaneous minimum cross-sectional area A_i, i.e.

$$\sigma = P/A_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.11)$$

A typical true stress - true strain curve is shown in Fig. 3-6. NB. The true stress–strain curve reaches a maximum only at fracture.

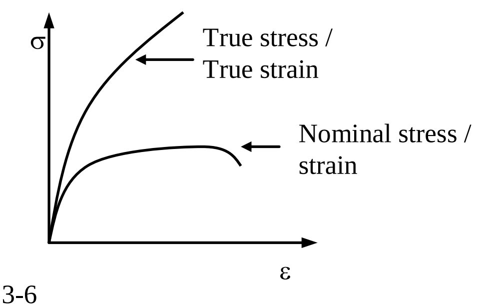


Fig. 3-6

True strain: It is the integral of the incremental change in length, *dl*, to the instantaneous length, l_i, of the sample (measured in absence of the stress) i.e.

$$\varepsilon_T = \int_{l_o}^{l_i} \frac{dl}{l} = \log\left(\frac{l_i}{l_o}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3.12)$$

3.4.3 Shear modulus

A force applied parallel to one of the planes of the specimen produces a shear deformation (Fig. 6.4). The shear stress (τ) ∝ shear strain (γ) i.e.

$$\tau = G\gamma \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(3.13)$$

where G is the shear modulus.

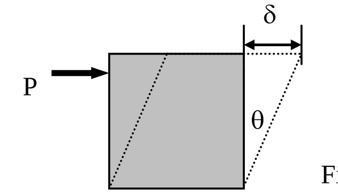


Fig. 6.4

3.4.4 Ductility and Brittleness:

As regards mechanical properties, materials divide sharply into two groups, *ductile* and *brittle*. Ductile materials (e.g. metals deform plastically before fracture) and as such, ductility is that property of a material which enables it to be drawn out into a thin wire e.g. mild steel. Brittle materials, notably ceramics, and glasses fracture without much plastic flaw. Ductility can be measured as:

(i) percentage elongation i.e., $\%\ Elongation = \dfrac{L - L_o}{L_o} \times 100\%$

(ii) % Reduction in area (determined at the necked region) i.e.,

$$\%\ R.A. = \frac{d_o^2 - d^2}{d_o^2} \times 100\%$$

where d_o is the original diameter and d is the reduced diameter

A convenient dividing line is to call materials ductile if they neck before fracturing, and brittle if they fracture without any necking.

Malleability:
It is the property/ability of a material to be flattened into sheets without cracking by pressing, rolling, hammering etc.

3.5 Compressive Strength

In compressive test, the uniaxial load is applied compressively (Fig. 3.7).

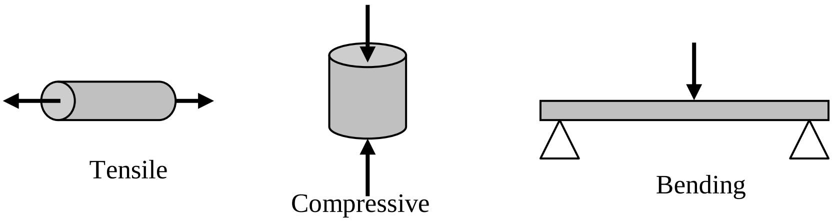


Fig. 3.7. Tensile, compressive and bending tests for materials

Compressive tests are particularly useful for evaluating strength properties of brittle materials used in compression e.g. cast iron, concrete, brick, ceramics and wood. Brittle materials usually contain microscopic cracks that result in their premature failure in tension and as such, the fracture strength of brittle materials is an order of magnitude higher in compression than in tension (Fig. 3-8).

In triaxial compression, the compressive stress or hydraulic pressure (P_h) is $\propto$ to compressive strain (volume strain $\Delta V/V$) i.e.,

$$P_h = K\left(\frac{\Delta V}{V}\right) \qquad \text{……………………….…(3.14)}$$

where K is the bulk modulus (compressive modulus of elasticity).

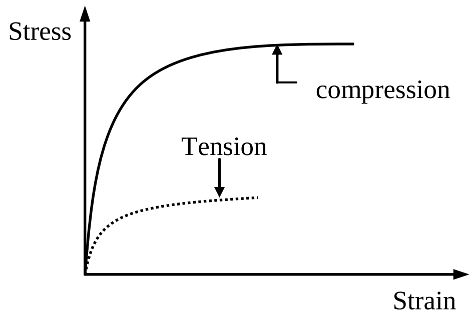


Fig. 3-8. Concrete in Compression and Tension

3.6 The Energy Capacity of a Material

The ability of a material to absorb and store energy is related directly to its shock-resistance and toughness. When the atomic bonds of a material are strained by an external force, elastic energy is stored in the body and can be fully recovered as mechanical work done by the body if applied force is removed. For example, suppose a material cube is strained by a force F as shown in Fig. 3-9.

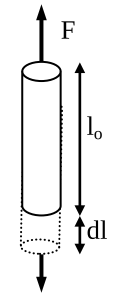


Fig. 3-9

Work done by external force is
W = *Av. Force (Extension)* = ½ *F.dl*

But Stress (σ) = Force/Area = F/l_o^2
Strain (ε) $= dl/l_o$

Therefore, $W = ½(\sigma\, l_o^2)(\varepsilon l_o)$

$= ½\, \sigma\varepsilon(l_o^3)$

Thus work done per unit volume = ½ $\sigma\varepsilon$ = Strain Energy density

$\Rightarrow$ Strain energy density = $\frac{1}{2}\sigma\varepsilon = \frac{\sigma^2}{2E} = \frac{1}{2}\varepsilon^2 E$ ………....(3.15)

Similarly, for shearing stress, strain energy density

$$U = \tfrac{1}{2}\tau\gamma = \frac{\tau^2}{2G} = \tfrac{1}{2}\gamma^2 G \quad \ldots\ldots\ldots\ldots(3.16)$$

And for hydrostatic pressure, strain energy density

$$U = \tfrac{1}{2}P\Theta = \frac{P^2}{2K} = \tfrac{1}{2}\Theta^2 K \quad \ldots\ldots\ldots\ldots(3.17)$$

where $\Theta = \Delta V/V$

Strain energy density stored upto the proportional limit ($\sigma = \sigma_p$ and $\varepsilon = \varepsilon_p$ in above Eqns.) is the *modulus of elastic resilience. Resilience is the ability of a material to store energy elastically*. Materials with high resilience are used for springs.

Worked Examples

1. A steel piano wire of length 1.8m and radius 0.3 mm is subjected to a tension of 70 N by a weight attached to its lower end. By how much does this wire stretch in excess of its initial length ? (Young's modulus for steel is 22×10^{10} Nm^{-2}).

Solution

Cross-sect area A = $\pi r^2 = 2.8 \times 10^{-7}$ m^2. And

$\frac{F}{A} = \frac{70\text{ N}}{2.8\times10^{-7}\text{ m}^2} = 2.5\times10^8\text{ Nm}^{-2}$. By Hooke's law ; $\frac{F}{A} = Y\frac{e}{l}$

$\Rightarrow \frac{e}{l} = \frac{1}{Y}\frac{F}{A} = \frac{1}{22\times10^{10}\text{ Nm}^{-2}}\times 2.5\times10^8\text{ Nm}^{-2} = 1.1\times10^{-3}$

$\therefore$ e = 1.8 m × 1.1×10^{-3} = 2×10^{-3} m = 2.0 mm

Questions for discussion

1. Explain briefly what is meant by fatigue fracture. Discuss some situations in which this type of fracture is likely to occur.

2. Explain in brief why girders have the familiar H-shaped cross-section, and why bicycle frames are made from *hollow tubes*.

3. Many buildings are usually made of brittle materials (bricks, stones, cast iron and glass). Briefly explain why.

4. Describe the changes, which take place when a wire is subjected to a steady increasing tension. Include in your description a sketch of tension vs. extension for a ductile material such as copper and a brittle material such as cast iron.

5. Define stress and strain and explain why these quantities are useful in studying the elastic behaviour of materials

TUTORIAL 3.2

1. A load of 1.5 kg is attached to a wire of length 3 m and diameter 0.46 mm. If the extension produced is 2.0 mm, calculate the stress, strain and hence determine the Young's modulus for the wire.

2. A wire of length 5m and of uniform circular cross-section of radius 1mm is extended by 1.5mm when subjected to a uniform tension of 100N. Calculate from first principles the strain energy per unit volume assuming that the deformation obeys Hooke's law.

3. A light steel wire, 1.5m long and cross-sectional area 2.5mm^2 is supported horizontally between two points 1.5m apart. A load attached to its mid-point causes a vertical displacement of 6.0mm. Calculate
   (a) the extension of the wire
   (b) the tension in the wire
   (c) the magnitude of the load. (Young's modulus of steel = 22.0×10^{10} Nm^{-2}).

4. A rod of original length 1.2m and area of cross section 1.5×10^{-4} m^2 is extended by 3.0mm when the stretching tension is 6.0N. Calculate the energy density of the stretched rod (the elastic P.E per unit volume).

5. A spring is extended by 30 mm when a force of 1.5N is applied to it. Calculate
   (i) the energy stored in the spring when hanging vertically supporting a mass of 0.20 Kg if the spring was unstretched before applying the mass
   (ii) the loss in potential energy and explain why these values differ.

6. Show that the energy stored in a wire of length L when it is extended by a length l is $\frac{1}{2}\left(\frac{El^2}{L^2}\right)$

   per unit volume where E is the Young's modulus of the material.

Suggested Further Reading

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Module Four: ROTATIONAL DYNAMICS

Rotational Motion about a fixed axis
Kinetic Energy of Rotation
Moment of Inertia
Work, Energy and Power
Angular Momentum
Angular Impulse

USEFUL RELATIONSHIPS

Rotational Dynamics

Linear (Tangential) Velocity,	$v = \frac{ds}{dt} = r\omega$
Tangential acceleration	$a_{\tan g} = \frac{dv}{dt} = r\alpha$
Angular Velocity,	$\omega = \frac{d\theta}{dt} = \frac{v}{r}$
Angular acceleration,	$\alpha = \frac{d\omega}{dt}$
Equations of motion,	$\omega = \omega_o + \alpha t$
	$\theta = \omega_o t + \frac{1}{2}\alpha t^2$
	$\omega^2 = \omega_o{}^2 + 2\alpha\theta$
Moment of Inertia,	$I = \sum_{i=1}^{n} M r_i^2$
Angular Momentum,	$\Omega = I\omega = r \times L$
Torque,	$\Im = \frac{d\Omega}{dt} = I\alpha = 2Fr$
Work,	$W = \Im\theta = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_o^2$
Power,	$P = \Im\frac{d\theta}{dt} = \Im\omega$
Kinetic Energy,	$E_k = \frac{1}{2}I\omega^2$
Angular Impulse,	$\Im dt = I\omega - I\omega_o$
Radius of Gyration,	$I = MK^2$

ROTATIONAL DYNAMICS

4.0 Rigid Bodies

A rigid body is a system of particles with a fixed shape in which parts of the body (particles) have fixed positions relative to one another e.g., a hammer. A rigid body can simultaneously undergo two kinds of motion i.e., it can change the position of its centre of mass (C.M) in space (translational motion) and/or it can change its orientation in space (rotational motion about some axis).

Assessment Objectives

- *Candidates should be able to*

(q) Define angular displacement, angular velocity and angular acceleration

(xiii) Derive and apply the equations of motion for objects moving with constant angular acceleration

(xiv) Deferentiate between tangential and centripetal acceleration

(xv) Define Moment of inertia and calculate the moment of inertia for various objects

(xvi) Derive the expressions for Work, Kinetic energy, and power in rotational motion and show the analogoue in these quantities with the same quantities in linear motion

(xvii) Define and apply the conservation of angular momemtum in real life situations.

4.1 Rotational Motion about a Fixed Axis

The simplest case of rotational motion of a rigid body is the rotation about a fixed axis e.g. a turntable, a roulette wheel or a revolving door. Fig. 4-1 shows a rigid body being rotated about an axis (into the paper) at point O.

If the body rotates through angle $\Delta\theta$ in time Δt, then the *angular velocity (ω)* is given as

$$\omega = \frac{\Delta\theta}{\Delta t} \quad or \quad = \frac{d\theta}{dt} \text{ rads}^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

Also since $\theta = s/r$,

$$\Rightarrow \quad \omega = \frac{s}{rt} = \frac{v}{r} \quad or \quad v = r\omega \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

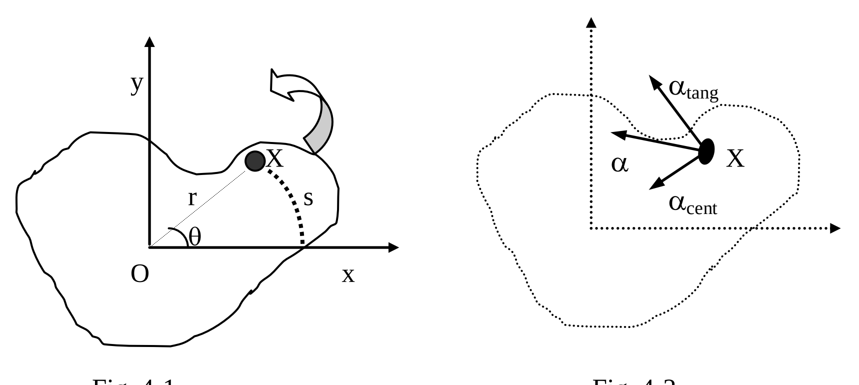


Fig. 4-1. Fig. 4-2.

- If ω is constant then the period (*T*) and the frequency (*f*) of rotation are given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3)$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.4)$$

- If ω changes, then the angular acceleration (α) can be given by

$$\alpha = \frac{d\omega}{dt} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.5)$$

- If the particle, X in the body moves through arc length *s*, then its translational speed is

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.6)$$

- Consequently, the *tangential* and *centripetal* (*normal*) acceleration of particle X are

$$\alpha_{\tan g} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.7)$$

$$\alpha_{cent} = \frac{v^2}{r} = r\omega^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.8)$$

The resultant acceleration (α) is the vector sum of α_{tang} and α_{cent} (See Fig. 4-2). **NB**. $v \propto r$, i.e., the further the particle from the axis of rotation, the faster it moves.

4.1.1 Motion with constant angular acceleration

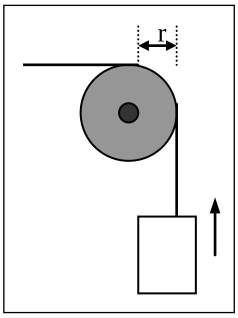
The equation describing rotational motion with constant angular acceleration are analogous to those describing translational motion with constant linear acceleration (with θ taking the place of ***x***, ω the place of ***v*** and α the place of ***a***) as shown below.

Translational motion	*Rotational motion*
$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$
$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$	$\omega = \omega_o + \alpha t$
$s = \boldsymbol{u}t + ½\,\boldsymbol{a}t^2$	$\theta = \omega_o t + ½\,\alpha t^2$
$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$	$\omega^2 = \omega_o^2 + 2\alpha\theta$

Worked Example

1. The cable supporting an elevator runs over a wheel of radius 0.36m without slipping. If the elevator ascends with an upward acceleration of $0.60ms^{-1}$, determine
   (i) the angular acceleration of the wheel
   (ii) the number of turns the wheel makes if this accelerated motion starts from rest and lasts 5.0s.

Solution



Acceleration of cable = a_{tang} of a point on the rim of the wheel i.e.,

$$a = a_{tang} = r\alpha$$

$$\Rightarrow \quad \alpha = a/r = 1.67 \text{ rad s}^{-2}$$

Using $\theta = \omega_o t + \frac{1}{2}\alpha t^2$, then angular displacement $\theta = 0 + \frac{1}{2}(1.67)(5^2) = 21$ rads.

But 1 revolution = 2π rads

$\therefore$ 21 rads = 3.3 revolutions

4.2 Kinetic Energy Of Rotation*: Moment Of Inertia*

Since a rigid body is a system of particles, the total kinetic energy of a rotating rigid body is the sum of the individual kinetic energies of all the particles. If the particles in the body have masses M_1, M_2, M_3 ……M_n and speeds V_1, V_2, V_3 ……V_n and their position vectors are r_1, r_2, r_3 ………r_n as shown in Fig. 4-3, then

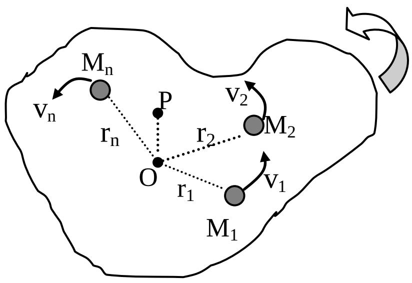


Fig. 4-3.

$$K.E. = \tfrac{1}{2}M_1v_1^2 + \tfrac{1}{2}M_2v_2^2 + \tfrac{1}{2}M_3v_3^2 ....+ \tfrac{1}{2}M_nv_n^2.$$

Since ω is the same for all particles, then $v_1 = r_1\omega,\quad v_2 = r_2\omega, ..v_n = r_n\omega$ *etc*.

$$\Rightarrow \quad K.E = \tfrac{1}{2}M_1r_1^2\omega^2 + \tfrac{1}{2}M_2r_2^2\omega^2 + \tfrac{1}{2}M_3r_3^2\omega^2 ....+ \tfrac{1}{2}M_nr_n^2\omega^2.$$

Or K.E. $=\frac{1}{2}I\omega^2$ ……………………………..………..………..(4.9)

where $I = M_1 r_1^2 + M_2 r_2^2 + M_3 r_3^2 ....+ M_n r_n^2 = \sum_{i=1}^{n} M r_i^2$ ………………...…(4.10)

Note: Eqn. (4.9) is reminiscent of K.E. $=\frac{1}{2}Mv^2$ in translational motion. *I* is *the moment of inertia* of the rotating object and it is *a measure of the resistance that a body offers to changes in its rotational motion* just as mass is a measure of the resistance that a body offers to changes in its translational motion.

The moment of inertia is thus an analogue of mass in linear motion and this analogue becomes clear from the similarities in the relation for momentum, force, impulse, energy and work as illustrated below. *I* is a scalar quantity (since its value about a given axis remains unchanged by reversing its direction of rotation about that axis i.e., *I* has no direction). *I* depends not only on mass of an object but also on the distribution of mass about the axis of rotation e.g., a wheel has a greater moment of inertia compared to a solid disc of the same mass.

NB. since an ordinary rigid body is not a discrete assembly of point masses but rather a continuous distribution of mass, the summation in equation (4.10) can be replaced by an integral. If the rigid body is continuos with uniform density ρ, then the element of mass, *dm*, in a volume element *dv*, is *dm* =ρ*dv*. Therefore

$$I = \sum_{i=1}^{n} M r_i^2 = \int r^2 dm = \rho \int r^2 dv \qquad ……………….(4.11)$$

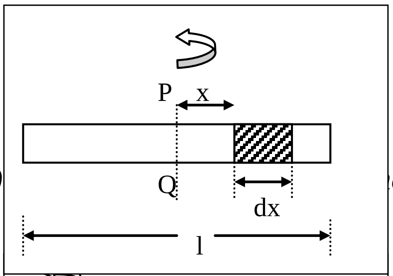
This integral can be performed quite readily for common simple shapes such as cylinders, spheres, and rectangular blocks.

4.2.1 Calculating the Moment of Inertia for various Objects

The moment of inertia (*I*) for different objects can be calculated by the same method used in calculating the centre of mass i.e. by subdividing the body into small volume elements and the moment of inertia contributed by each volume element summed up (using Eqn. 4.11) to give the overall *I* as illustrated below.

(i) Moment of Inertia of a Uniform Rod.

(a) About an axis through the middle

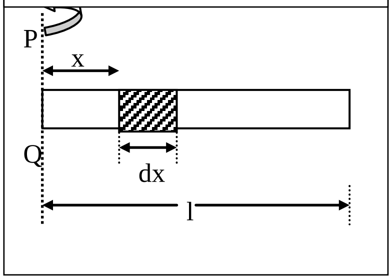


I for a small element *dx* about an axis PQ through its centre perpendicular to the length is $dI = \left(\frac{dx}{l}M\right)x^2$

Thus *I* for the whole rod is

$$I = \int dI = \int_{0}^{\frac{l}{2}}\left(\frac{dx}{l}M\right)x^2 = \frac{Ml^2}{12}$$

(b) *d*



In this case, *x* is measured from one end and *I* for a small element *dx* about an axis PQ through one end perpendicular to the length is

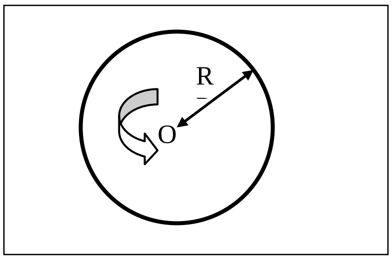
$$dI = \left(\frac{dx}{l}M\right)x^2$$

Thus *I* for the whole rod is

$$I = \int dI = \int_{0}^{l}\left(\frac{dx}{l}M\right)x^2 = \frac{Ml^2}{3}$$

- Similarly, it can be shown that:
  I of a disc of radius R about the axis of symmetry is = $\frac{1}{2}MR^2$
  I of a sphere of radius R about the diameter is = $\frac{2}{5}MR^2$

(c) Moment of inertia for ring



In a ring, every element is same distance from O. Thus for a ring of mass M, *I* about an axis through the centre perpendicular to the plane of the ring is

$$I = MR^2$$

4.2.2 Radius of Gyration

If the entire mass of a body is concentrated at a point such that the kinetic energy of rotation is the same as that of the body itself, then the distance of that point from the axis of rotation is the *radius of gyration* of the body about that axis. For example, From Fig. 4-3, if point P is the C.M., then

$$K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2}\sum mr^2\omega^2 = \frac{1}{2}MK^2\omega^2$$

where M is the total mass, K is the radius of gyration, and $MK^2 = \sum mr^2$.

Thus $K = \left(\sum_{i=1}^{n} r_i^2 \Big/ n \right)^{\frac{1}{2}}$ and

$$MK^2 = I \qquad \text{……………………………………………..(4.12)}$$

4.2.3 The Perpendicular Axis Theorem

Consider a laminar object in Fig. 4-4. If O-x and O-y are the two perpendicular axes in the plane of the lamina then, the moments of inertia of a particle X of mass m_1 about x and y-axes are

$$I_x = m_1 y_1^2 \qquad \text{and} \qquad I_y = m_1 x_1^2$$

The moments of inertia of the whole lamina about O-x and O-y are

$$I_x = \sum m_i y_i^2 \qquad \text{and} \qquad I_y = \sum m_i x_i^2$$

And the moment of inertia of the whole lamina about a perpendicular axes through O is

$$I = \sum mr^2 \qquad \text{but} \qquad r^2 = y^2 + x^2$$

$$\therefore \qquad I = I_x + I_y \qquad \text{………………………………………(4.13)}$$

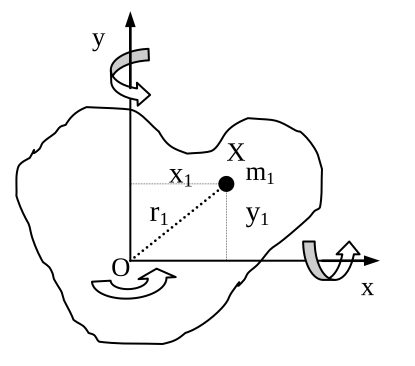


Fig. 4-4.

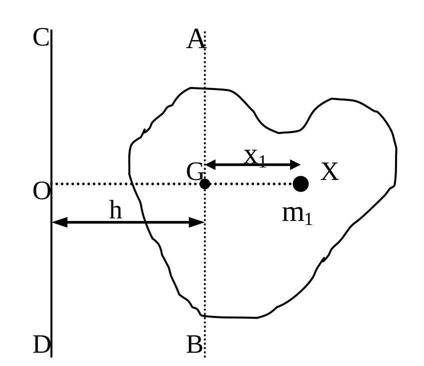


Fig. 4-5.

The moment of inertia of a plane lamina about an axes perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about the two axes at right angles to each other in its own plane intersecting each other at the point where the perpendicular axes passes through it [Theorem of perpendicular axes].

4.2.4 Parallel Axes Theorem

Consider a rigid body shown in Fig. 4-5. Let CD be an axis in the plane of the paper and AB be a parallel axis through G (centre of mass of the body). If h is the distance between the two axes then

Moment of inertia of mass m_1 about CD is

$$CD = m_1(x_1 + h)^2 = m_1x_1^2 + m_2h^2 + 2m_1x_1h\,.$$

Therefore I of the body about CD is

$$I = \sum m_1x_1^2 + \sum m_1h^2 + 2\sum m_1x_1h$$

If I_g is the moment of inertia of the body about AB then $I_g = \sum m_1x_1^2$. Therefore

$$I = I_g + \sum m_1h^2 + 2h\sum m_1x_1$$

But $\sum m_1x_1$ is the total moment of all particles about AB passing through G. Since the body is balanced at G, it follows that $\sum m_1x_1 = 0$. Therefore

$$I = I_g + Mh^2 \qquad \text{……………………….……….(4.14)}$$

where M is the mass of the body.

Thus the moment of inertia of a body about any axes is equal to the sum of its moment of inertia about a parallel axis through its centre of gravity (I_g) and the product of its mass and the square of the distance between the two axes [Theorem of the Parallel axes]. This theorem is useful when calculating the moment of inertia about some arbitrary axis, once $\boldsymbol{I_g}$ is known.

For example, we saw that for a uniform rod of length l, The moment of inertia $\boldsymbol{I}$ about its centre is given by $I_C = \frac{1}{12}ML^2$ while $\boldsymbol{I}$ for the same rod about an axis through one end is given by $I = \frac{1}{3}ML^2$

We note that $\frac{1}{3}ML^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$

Or $\quad I = I_C + Mh^2$

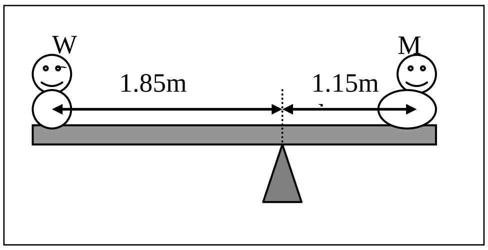
Where h =½L is the perpendicular distance between the two axes.

Worked Example

2. A 50-Kg woman and an 80kg man sit on a massless seesaw of length 3.0m. The seesaw rotates about a pivot placed at the centre of mass of the system. If the

(instantaneous) angular velocity of the seesaw is 0.4 rads^{-1}, calculate the kinetic energy.

Solution



$$I = M_w r_w^2 + M_m r_m^2$$
$$= 50(1.85)^2 + 80(1.15)^2$$
$$= 277 \text{ Kgm}^2$$

$$\text{K.E.} = \tfrac{1}{2} I\omega^2$$
$$= \tfrac{1}{2} M_w r_w^2 \omega^2 + \tfrac{1}{2} M_m r_m^2 \omega^2$$
$$= 22 \text{ J}$$

4.3 Work, Energy, Power and Momentum in Rotational Motion

4.3.1 Work and Energy

Consider a rigid body (Fig. 4-4) being acted upon by an external force F. The effective force required to rotates the object (and hence a particle X) through angle θ acts along the tangent to the circular path described by particle X and is $F \sin\theta$.

- If the particle moves through a displacement **s**, then work done is

$$W = (F \sin\theta)s = Fr\theta\ (\sin\theta);\quad \text{But } Fr \sin\theta = \text{Torque} = \Im$$

$$\Rightarrow \quad W = \Im\theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.15)$$

Eqn (4.15) is a general equation for the work done to rotate a rigid body. NB. This is analogous to $W = Fs$ in translational motion.

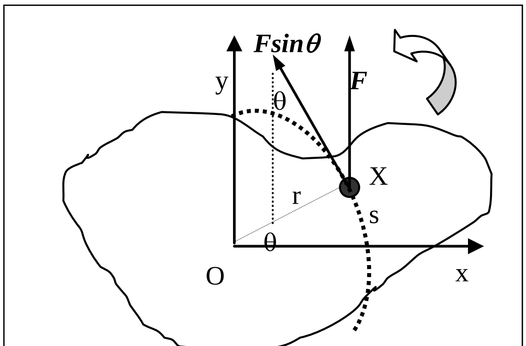


Fig. 4-4

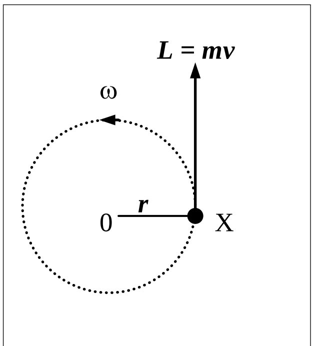


Fig. 4-4b

- Due to this work, the body acquires rotational K.E i.e., W = K.E. = $\frac{1}{2} I\omega^2$
- This K.E may change into P.E and vice versa. If the torque acting on the body is conservative, then the total energy is conserved i.e.,

$$\frac{1}{2} I\omega^2 + U = constant$$

Power

The power delivered by the torque is $P = \frac{\Delta W}{\Delta t} = \Im \frac{d\theta}{dt}$

$$\Rightarrow \qquad P = \Im\omega = \frac{dW}{dt} \qquad \text{………..……………….(4.16)}$$

Angular Impulse

The angular impulse is the product of the torque and the time for which it acts i.e.,

$$\Gamma = \int_{t_1}^{t^2} \Im dt = \Omega_2 - \Omega_1 = I(\omega_2 - \omega_1) \qquad \text{….……………….…..(4.17)}$$

where $\boldsymbol{\Omega_i}$ is the angular momentum

Angular Momentum (Ω)

The angular momentum Ω of a particle about a fixed point is the moment of its linear momentum about that point. For example, in Fig. 4-6b, the angular momentum of particle X describing circular motion with uniform angular velocity ω and linear velocity ***v*** (= ***r***ω) is

$$\boldsymbol{\Omega} = \boldsymbol{r} \times \boldsymbol{L} = \boldsymbol{r}\, L\sin\theta = m\,(\boldsymbol{r} \times \boldsymbol{v}) = M\boldsymbol{r}^2\omega \qquad \text{…………….…..…(4.18)}$$

The direction of $\boldsymbol{\Omega}$ is perpendicular to both ***L*** and ***v***.

For a rigid body (Fig. 4-6), $\boldsymbol{\Omega}$ is the sum of the individual angular momenta of all the particles i.e.,

$$\Omega = \sum_{i=1}^{n} m_i r_i^2 \omega \;\; = \boldsymbol{I}\omega \qquad \text{……………..…………….………(4.19)}$$

Torque as Time Rate of Angular Momentum

From the definition $\boldsymbol{\Omega} = \boldsymbol{r} \times \boldsymbol{L}$, Differentiating *w.r.t* time we get

$$\frac{d\Omega}{dt} = \frac{d}{dt}(r \times L) = \frac{dr}{dt} \times L \;+\; r \times \frac{dL}{dt}$$

But $\frac{dr}{dt} \times \mathrm{L} = v \times Mv = M(v \times v) = 0$ and $\frac{dL}{dt} = F$

$$\therefore \qquad \frac{d\Omega}{dt} = r \times F = \Im \qquad \text{……… ………………..…..….(4.20)}$$

Thus torque on a rigid body is the rate of change of angular momentum*.

Further, since $\Omega = I\omega$, then

$$\Im = I\alpha \qquad \text{…………………………………..…(4.21)}$$

*Alternative approach

- From Eqn. (3.4), Torque $\Im = \boldsymbol{r} \times \boldsymbol{F} = \boldsymbol{r}$ (Fsin θ). But $F = dL/dt;\ \theta = 90^o$

$$\Rightarrow \qquad \Im = \frac{d}{dt}\{\boldsymbol{r} \times \boldsymbol{L}\} = \frac{d\Omega}{dt} \qquad \text{…………………..…..………..…(a)}$$

Also since $\boldsymbol{L} = M\boldsymbol{v} = M\boldsymbol{r}\omega$ and $\boldsymbol{v} = \boldsymbol{r}\omega$

$$\Rightarrow \qquad \Im = \frac{d}{dt}\{M\boldsymbol{r}^2\omega\} = \frac{d}{dt}(I\omega)$$

$$\Rightarrow \qquad \Im = I\alpha \qquad \text{…………………………….…………..….(b)}$$

If follows from (a) that the angular momentum is the moment of the linear momentum i.e., proof of Eqn. 4.18.

4.3.2 Conservation of Angular Momentum

From Eqn (4.20) it follows that if no external torque is applied to a body or system of bodies, then, its angular momentum remains constant. This is the *Principle of conservation of angular momentum*. i.e.

$$\text{if } \Im = \frac{d\Omega}{dt} = 0 \Rightarrow \quad \Omega = const. \qquad \text{…….…………………….……………(4.22)}$$

A good example of conservation of angular momentum is applicable in figure scatting when performing a pirouette. In order for a scatter spinning with angular velocity ω_i to increase his/her speed, conservation of angular momentum ($l_i\omega_i = l_f\omega_f = const$) demands for a decrease in the moment of inertia. Thus he/she has to clasps his/her arms around himself/herself to decrease *I*. NB. the greater the moment of inertia of a body, the greater is the couple required to produce a given angular acceleration.

The following are some of the analogous between parameters in translational motion and rotational motion. (θ taking the place of ***s***, ω the place of ***v*** and α the place of ***a*** and ***I*** the place of M).

Translational motion	*Rotational motion*
$F = Ma$	$\Im = I\alpha$
$W = Fs$	$W = \Im\theta$
$P = FV$	$P = \Im\omega$
$K.E. = \frac{1}{2}Mv^2$	$K.E. = \frac{1}{2}I\omega^2$
$L = MV$	$\Omega = I\omega$
$I = M\,dv$	$\Gamma = Id\omega$

Worked Examples

1. A particle of mass *m* moves on a path given by the equation $r = a\cos\omega t\, i + b\sin\omega t j$. Calculate the torque and angular momentum about the origin.

   Solution:
   Given $r = a\cos\omega t\, i + b\sin\omega t j$, therefore
   $\mathbf{v} = \dot{\mathbf{r}} = \omega(-a\sin\omega t\, i + b\cos\omega t j)$ and $a = \dot{\mathbf{v}} = -\omega^2\mathbf{r}$

   $\Rightarrow$ $\boldsymbol{F = Ma = -M\omega^2 r}$ i.e., linear restoring force.

   $\Rightarrow$ Torque $\boldsymbol{\Im} = \mathrm{r} \times \mathrm{F} = -M\omega^2(\mathbf{r}\times\mathbf{r}) = 0$

   $\Rightarrow$ Angular momentum $\Omega = M(r\times v) = M\omega ab$ (in +ve z-direction).

2. A grind stone weighing 40 Kg has a radius of 1.2m. Starting from rest it acquires a speed of 150 *r.p.m* in 12 sec. Calculate the torque acting on it.

   Solution:
   $I = \frac{1}{2}MR^2 = 28.8\, Kgm^2$ and angular acceleration is $\alpha = \frac{2\pi\times150}{60\times12} = \frac{5\pi}{12} rads^{-1}$

   $\Rightarrow$ $\Im = I\alpha = 37.7\ Nm$

3. A circular disc of mass *m* and radius *r* is set rolling on a table. If ω is the angular velocity, show that its total energy is given by $\frac{3}{4}Mr^2\omega^2$.

   Solution
   When the disc rolls, a point on its circumference rotates about an axes passing through its centre and perpendicular to its plane. In addition, the point moves forward. Hence, the disc posses both angular and linear velocity and consequently both linear and rotational energy.

   K. E. due to linear motion = $\frac{1}{2}Mv^2$

   K. E. due to rotational motion = $\frac{1}{2}I\omega^2$ But $I = \frac{1}{2}Mr^2$ and v = rω.

   Hence total K.E. = $\frac{1}{2}Mv^2 + \frac{1}{4}Mr^2\omega^2 = \frac{3}{4}Mr^2\omega^2$.

TUTORIAL 4.1

1. Calculate the rotational kinetic energy of the earth. Treat the earth as a rigid sphere of uniform density with a radius of 6.37×10^6m and mass 5.89×10^{24} Kg and ignore the orbital motion of the earth round the sun.

2. When is the minute hand of a watch directly over the hour hand? How frequently are the hour, minute and the sweep second hands all coincident?

3. A car has wheels of radius 0.30 m and is travelling at 36 ms^{-1} in a straight line.
   (a) Calculate the angular speed of the wheels about the axle
   (b) Describe the path of a point on the rim.
   (c) The wheels now describe 40 revolutions as the car is being brought to rest. Calculate the wheels' angular acceleration and the distance covered during braking

4. A meter stick initially standing vertically on the floor falls over. Find the angular velocity with which the stick hits the floor assuming the end of the stick in contact with the floor does not slip.

5. A particle of mass 2 units moves in a force field depending on time *t* given by $F = 24t^2 i + (36t - 16) j - 12tk$. If the position of the particle at time *t* is given by $r = (t^2 + 6t + 3)i + (3t^3 - 4t^2 + 15t) j + (4 - t^3 - 8t)k$, determine
   (a) the torque and
   (b) the angular momentum about the origin for the particle at any time *t*.

6. A uniform sphere rolls down a plane inclined at an angle α. Show that the acceleration of the sphere is $(5/7)\ g \sin \alpha$.

7. A particle moves in a force field given by $\boldsymbol{F} = \boldsymbol{r}^2 R$ where $\boldsymbol{r}$ is the position vector of the particle. Prove that the angular momentum of the particle is conserved.

8. Calculate the moment of inertia of a uniform metre rule of mass 0.10 Kg about a transverse axis through
   (i) the 0.50 m mark, and hence
   (ii) the 0.90m mark. Neglect the ruler's width.

9. A skater has a moment of inertia 5.0 Kgm^2 when she has both arms and one leg outstretched. She is turning at 4.0 rad s^{-1}. When she draws her arms and leg inwards her moment of inertia becomes 0.60 Kg m^2. Calculate
   (a) Her final angular speed
   (b) The change in her rotational k.e. How do you account for (b)?

10. An impulse of 40 Ns is transmitted to the rim of a spinning wheel. If the wheel is a flat uniform disc of mass 24 Kg and radius 0.60m, calculate
    (a) the angular impulse transmitted to the wheel, and
    (b) the change in angular speed.

11. An engine has rotating parts of mass 18 Kg and radius of gyration 0.12m. They reach an angular speed of 0.20 K rad s^{-1} from rest in 4.0s. Calculate
    (a) the angular momentum and kinetic energy at this angular speed.
    (b) the torque and average power needed to reach it.

Module Five: FLUID STATICS & DYNAMICS

Fluid Statics
Fluid Dynamics
Viscosity

USEFUL RELATIONSHIPS

Fluid Statics

Pressure in a Fluid, $P = \frac{F}{A}$

Hydrostatic Pressure, $P - P_o = \rho g h$

Pascal's Principle, $\frac{F_1}{A_1} = \frac{F_2}{A_2} = P$

Fluid Dynamics

Continuity Equation, $A_1 v_1 = A_2 v_2$

Bernouilli's Equation, $P + \rho g h + \frac{1}{2}\rho v^2 = constant$

Viscous Friction, $F = \frac{\eta V A}{h}$

Poiseuille's Equation, $\frac{\Delta V}{\Delta t} = \frac{\pi r^4 (P_1 - P_2)}{8\eta l}$

Fluids play a vital role in many aspects of everyday life. We drink them, breath them, swim in them. They circulate through our bodies and control the weather. Airplanes fly through them; ships float in them. *A fluid is any substance that can flow or it is a system of particles loosely held together by their own cohesive forces (e.g. in liquids) or by the restraining forces exerted by the walls of a container (e.g. in gases).* Both liquids and gases are fluids. Notably, liquids are incompressible or nearly so (i.e. they have a constant volume), whereas gases are compressible.

Unlike in a rigid body where particles maintain fixed positions, in fluids, particles are loosely held and can wonder about within the volume of a fluid. Throughout most of this chapter, we will neglect the *viscosity* of fluids. *Viscosity is an internal friction or stickiness within the fluid that offers resistance to its flow*. For instance, honey is a fluid of high viscosity, whereas water is a fluid of fairly low viscosity. Viscous friction (viscosity) increases with speed of flow. *A perfect fluid is one in which there is no internal friction within the fluid*.

Assessment Objectives

- *Candidates should be able to*

(r) *Define the terms density, Pressure*
(ii) *Derive and use the expression* $P = \rho gh$
(iii) *Understand the origin of the upthrust acting on a body in a fluid*
(iv) *Appreciate that pressure differences can result from different rates of flow of a fluid (the Bernouilli effect) and explain the lift force which results from the Bernouilli effect of air moving over aerofoils*
(v) *Explain the transmission of pressure by fluids (Pascal's Principle)*

5.1 Fluid Statics

It is the study of fluids in equilibrium situations and is based on Newton's 1^{st} and 2^{nd} law. We shall explore concepts of pressure, buoyancy, and surface tension.

5.1.1 Pressure in a static fluid

A fluid is best described in terms of its density, velocity of flow and pressure. The density (ρ) of a fluid depends appreciably on temperature, and only slightly on pressure e.g., the density of water is maximum at 4^o C.

The average pressure (P) at a point in a static fluid (liquid or gas) is the normal force (F) per unit area on a very small area around the point i.e.,

$$P = \lim_{\delta A \to 0}\left(\frac{\delta F}{\delta A}\right) = \frac{F}{A} \qquad \text{……..………...…………..(5.1)}$$

P is a scalar since it acts in all directions and it increases with depth e.g., consider a fluid in a container of base area A and height h as shown in Fig. 5-1. The force on the base of the cube due to the fluid will be equal to the weight of the fluid. Thus pressure on area A is

$$P = \frac{F}{A} = \frac{Ah\rho g}{A} = h\rho g \quad (Nm^{-2}) \qquad \text{.......................(5.2)}$$

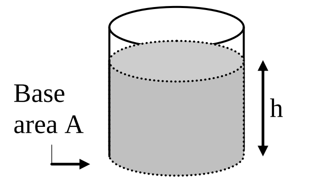


Fig. 5-1

The volume of fluid = Ah
Mass of fluid = $Ah\rho$

$\therefore$ Wt of fluid = $Ah\rho g$ = Force

where ρ is the density of the fluid.

Pressure is the same at all points on the same horizontal level in a fluid and it depends on the density (ρ) of the fluid. Less dense substances float on more dense ones. (Units of Pressure: $1 Nm^{-2} = 1 Pa$; $1 \text{ Atm} = 1.01 \times 10^5 Nm^{-2}$).

5.1.2 Archimede's Principle

Consider an object immersed in a fluid as shown in Fig. 5-2. Since pressure increases with depth, the pressure (P_A) due to the fluid at the bottom of the object is greater than the pressure (P_B) at the top (i.e., $P_A > P_B$)

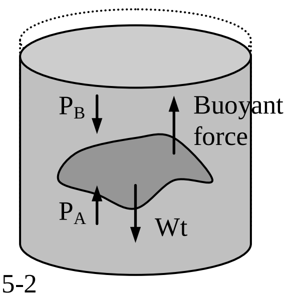


Fig. 5-2

The resultant upward (buoyant force) is

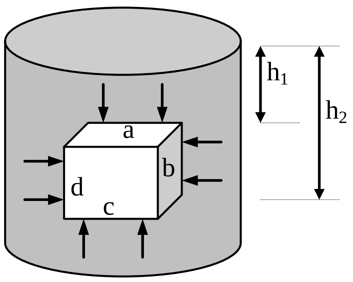
$P_A - P_B \Rightarrow$ Buoyant force (F_B)

When $F_B >$ Wt, the object will rise
When $F_B <$ Wt, the object will sink
When F_B = Wt, the object is at equilibrium and is partially submerged.

According to *Archimede's principle, the buoyant force (upthrust) on an immersed object has the same magnitude as the weight of the fluid displaced by the object*. This principle is widely applicable in submarines.

Prove of Archimede's Principle

Consider a solid of cross sectional area *A* immersed in a fluid (Fig. A). The resultant horizontal force on sides *b* and *d* is zero since they cancel out.



Upward force on side *c* = $h_2\rho gA$

Downward force on side *a* = $h_1\rho gA$

Resultant upward force = $h_2\rho gA - h_1\rho gA$

$= (h_2 - h_1)\rho gA$

= Upthrust

But $(h_2 - h_1)A$ = volume of solid = V

$\therefore$ Upthrust = $V\rho g$ = Mg

= wt of fluid displaced

Figure A

5.1.3 Pascal’s Principle

When the flow velocity of a fluid is everywhere zero, the fluid is in static equilibrium e.g., air in a closed room. In such a fluid, the pressure at all points within the fluid is the same. This uniformity of pressure throughout a static fluid implies that a pressure change to one part of the fluid will be transmitted undiminished (without change) to every portion of the fluid (see Eqn. 5.3) and to the walls of the containing vessel [*Pascal's principle*]. This principle finds widespread application in hydraulic systems (brakes, jerks, etc.) where a large resultant force is produced from a small-applied force as shown in Fig. 5-3.

$$\Rightarrow \qquad P = \frac{F_1}{A_1} = \frac{F_2}{A_2} = \frac{F_3}{A_3} \qquad \text{……………………………………..(5.3)}$$

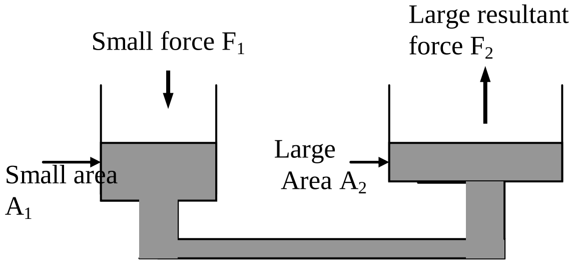


Fig. 5-3

Manometers

Since Pressure increases with depth (or height), several instruments measuring pressure (manometers) make use of the height of a liquid in a column of glass e.g. mercury. Manometers (pressure gauges) such as used for automobile tires read overpressure i.e., excess pressure over atmospheric pressure. For example, If p is the pressure to be measured, then

$$p = P + h\rho g \qquad \text{.......................................(5.4)}$$

where P is the atmospheric pressure and h is the height of the liquid column balanced by the pressure to be measured. The amount by which p exceeds atmospheric pressure equals pressure due to the column of liquid *i.e.*, $h\rho g$.

Since ρ and g are constant for particular conditions, it is convenient to state pressure as a number of *mm* or *cm* of water or mercury. To measure absolute pressure, two tubes can be used with one of the tubes closed. h then reads pressure directly. This principle is used in the mercury barometer, where the short limb is replaced by a reservoir of mercury. Sphygmomanometers used to measure cardiac pressure operate on similar principle.

5.2 Fluid Dynamics

It is the study of fluids in motion and is the most complex branches of mechanics.

(a) Incompressible steady flow: Streamlines

Consider the motion of a small element of the fluid flowing in a pipe [Fig. 5-2]. If with time the velocity at every point in the fluid remains constant in magnitude and direction, then the flow is a *steady flow*. Consequently, if each particle follows exactly the same path and has the same velocity as its predecessor, then the liquid has an orderly or *streamline (Laminar) flow*.

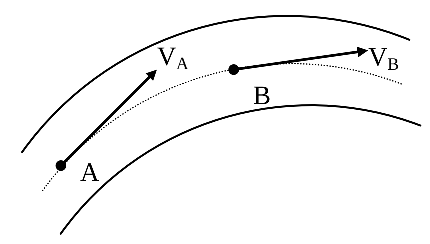


Fig. 5-4

A *streamline* is a line along which a fluid element moves when the flow is steady. The velocity of flow of the fluid at any point is a tangent on the streamline. In streamline flow, fluid elements cannot cross streamlines i.e., the layers slide over each other and dont intermix.

When the velocity of flow is greater than a critical value (critical velocity, V_c), different layers intermix and the flow becomes *turbulent* with eddies and whirlpools in the motion and the paths as well as the velocities of the particles continuously changes.

Notably, the flow of a fluid along a stream tube (i.e., a tube bounded by streamlines) will be fastest where the tube is narrowest. For example, from Fig. 5.5, the velocity of water at point B is greater than that at point A i.e., the magnitude of velocity is proportional to the density of streamlines. If the velocities and cross-sectional areas at ends A and B are v_1 and A_1 and v_2 and A_2 respectively (see Fig. 5.5), then

Volume of fluid passing through A_1 in time $\Delta t = v_1 A_1 \Delta t$
Volume of fluid passing through A_2 in time $\Delta t = v_2 A_2 \Delta t$

But vol. of fluid through A_1 in time Δt = Vol. of fluid through A_2 in time Δt. Thus:

$$v_1 A_1 \Delta t = v_2 A_2 \Delta t$$

Or $A_1 v_1 = A_2 v_2$ [Continuity Equation] ……..………(5.5)

NB. Since velocity (v) $\propto \frac{1}{A}$, and the density of streamline is proportional to $\left(\frac{No. of\ streamlines (N)}{Cross\ sectional\ area (A)}\right)$, then $v \propto$ density of streamline at a given point.

5.2.1 Bernoulli's Principle

When moving fluid encounters an obstacle that slows down its motion, the fluid exerts an extra pressure on the obstacle. You can feel the push of this extra pressure if you stand in a strong wind, or if you put a hand out of the window of a speeding car. The pressure changes that occur when a fluid flows round obstacles or pipes of varying cross-sections can be calculated by exploiting the conservation law for mechanical energy. The energy of a fluid in motion at any point consists of the following three forms

(i) Potential energy P.E. = mgh or ρgh per unit volume
(ii) Kinetic energy, K.E. = $½mv^2$ or $½\rho v^2$ per unit volume
(iii) Pressure energy which is the work done to move a liquid (= PV) there P is the pressure at a point and V is the volume of the fluid passing through that point per unit time i.e., $W = Fh = Pah = PV$.

For a special case of an incompressible fluid without viscosity (i.e., laminar flow), this conservation law is termed *Bernoulli's principle*. When a fluid is forced through a constriction in a tube where forward speed is increased, the pressure exerted sideways is decreased. In the case of an incompressible fluid flowing along a non-uniform pipe (Fig. 5-5), the pressure at both ends A and B of the pipe exert forces and do some work on the fluid. If the cross-sectional area at end A is A_1 and velocity of fluid is v_1, then the work done by the pressure P_1 in moving the fluid through length l_1 is

W_1 = Force (distance) = $A_1 P_1 l_1 = P_1 V_1$
where $V_1 = A_1 l_1$ = Volume of fluid between P and R

Likewise, work done by pressure P_2 at end B is

$W_2 = - A_2 P_2 l_2 = - P_2 V_2$ (-ve since force is opposite displacement)

Net work done $W = W_2 + W_1 = (P_1 V_1 - P_2 V_2)$. But $V_1 = V_2 = V$

$\Rightarrow \quad W = (P_1 - P_2)V$ ……………………..…....….(a)

Change in both Kinetic and Potential energy between ends A and B is given by

$\Delta K.E = \frac{1}{2}Mv_2^2 - \frac{1}{2}Mv_1^2$ ……….…………………………….…….(b)

and $\quad \Delta P.E = Mgh_2 - Mgh_1$ …………………………….….………..(c)

where h_1 and h_2 are the heights of the centers of fluid masses above the reference surface, v_1 and v_2 are the velocities of the fluid at ends A and B.

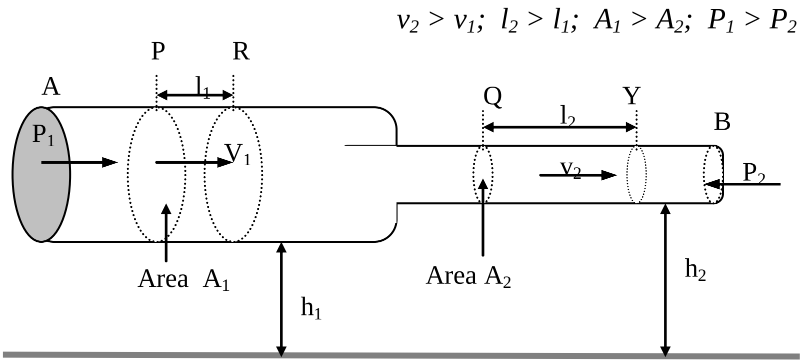


Fig. 5-5

Since W = Δ K.E. + Δ P.E.

$\Rightarrow \quad (P_1 - P_2)V = Mg(h_2 - h_1) + \frac{1}{2}M(v_2^2 - v_1^2)$, but $V = m/\rho$,

$\Rightarrow \quad P_1 - P_2 = \rho g h_2 - \rho g h_1 + \frac{1}{2}\rho(v_2^2 - v_1^2)$

$\Rightarrow \quad P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$

or $\quad P + \rho g h + \frac{1}{2}\rho v^2 =$ constant, [Bernoulli's Equation] …………...(5.6)

Thus for any two points on a streamline, the sums of K.E and P.E per unit volume and pressure are the same. NB. Bernoulli's equation is not valid in a compressible fluid (e.g. gas), a viscous fluid (with internal friction within the fluid) or in a flow of a fluid through a pump or turbine since these add or remove energy to the fluid.

5.2.2 Applications of Bernoulli's Principle

(a) *Aircraft wings*

According to Equation (5.6), in a region where the pressure (ρgh) within a fluid is uniform, the pressure (P) along a given streamline must decrease when velocity (v) increases. This concept plays a significant role in the design of aircraft wings (Fig. 5-6) etc.

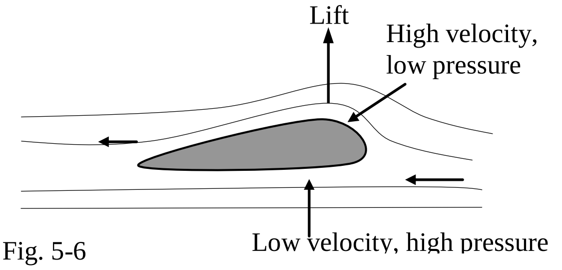


Fig. 5-6

The resultant lift supports the airplane in flight

(b) Sailboats

In a sailboat (Fig. 5-7), a similar lift acts on the sail (similar to wing of airplane) and on the keel of the sailboat.

The wind streaming over the sail generates a lift at right angles to the sail while the water streaming over the keel generates a lift force at right angles to the keel.

The resultant force (on the sail and keel) is a propulsive force, roughly parallel to the centerline of the sailboat which permit the sailboat to move upwind, at an angle as close as 40^{o} or 45^{o} relative to the wind direction.

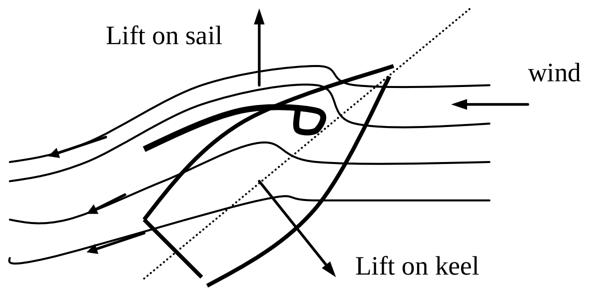


Fig. 5-7. Lift forces on the sail and keel of a sailboat sailing upwind.

(c) *The Sprayer*

Figure 5-8 shows a schematic diagram of a paint sprayer. One end of the tube A is connected at the end of tube B and the other end is immersed into the paint to be sprayed.

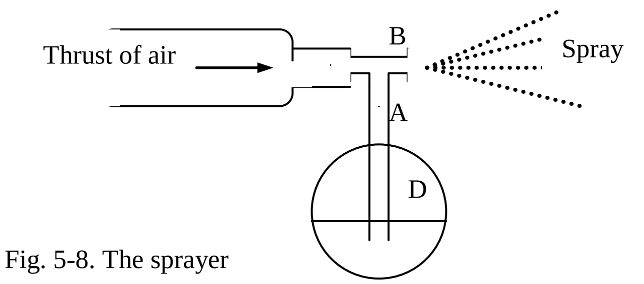


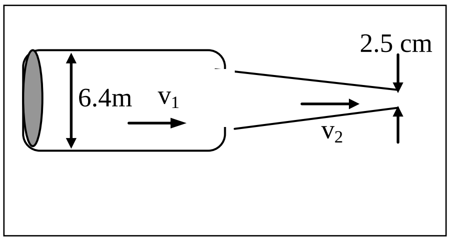
Fig. 5-8. The sprayer

The forward thrust of air produces a stream of air past the end of tube B. The pressure on the liquid in tube A is consequently reduced and atmospheric pressure acting on the surface of the liquid in D forces liquid up into tube A from which it is carried away by the stream of air constituting a spray (fine particles of paint).

Worked examples

1. The pressure in a firehose of diameter 6.4 cm is 3.5×10^5 Nm^{-2}. The firehose ends in a metal strip of diameter 2.5cm. Determine the velocities and pressures of the water in the tip.

   Solution



From continuity equation,

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = 26.2ms^{-1}$$

With $h_1 = h_2$, Bernoulli's equation gives

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad P_1 = 1.4 \times 10^4\,\text{Nm}^{-2}$$

2. Obtain an estimate for the velocity of emergence of a liquid from a hole in the side of a vessel 10 cm below the liquid surface.

   Solution
   At surface of liquid, P_1 = atmospheric pressure P, $h_1 = h$, $v_1 = 0$ assuming a wide vessel, *i.e.*rate of fall of surface is negligible.

   At the hole, P_2 = pressure of air into which liquid emerges = P, $h_2 = 0$, $v_2 = v$

   Substituting in Bernoulli's equation, $p_1 + h_1\rho g + \frac{1}{2}\rho v_1^2 = p_2 + h_2\rho g + \frac{1}{2}\rho v_2^2$

   $$\Rightarrow \quad h\rho g = \frac{1}{2}\rho v^2 \quad \text{or} \quad \mathbf{v} = \sqrt{2gh} = 1.1\ \text{ms}^{-1}.$$

   i.e. the P.E. lost by a unit volume of liquid (mass ρ) in falling from the surface to depth *h* is changed to K.E.

The velocity of emergence is given by $v^2 = 2gh$ and is the same as the vertical velocity acquired in free fall, a statement called *Torricelli's theorem*.

5.3 Viscosity

When a fluid flows through a pipe or conduit, the layers of the fluid adjacent to the walls stick to the walls thereby causing a viscous drag between these layers and the adjacent layers of the fluid. This tends to stop the flow of the fluid.

If the motion of a liquid over a horizontal solid surface is slow and steady, its layer in contact with the solid surface is stationary (zero velocity) while the velocity of any other layer above this one is faster and is proportional to its distance from the stationary layer and is maximum for the topmost layer. Thus, any two layers tend to oppose the flow of the layer immediately adjacent.

- *The property by virtue of which a liquid opposes relative motion between its different layers is called viscosity* **OR** *Viscosity is the internal friction in a fluid or it is the opposition set up by a fluid to shear*.

As a measure of viscous resistance, consider two plane parallel layers RS and PQ (Fig. 5-9) of liquid separated by a very small distance δy and having velocities $\boldsymbol{v} + \boldsymbol{\delta v}$ and $\boldsymbol{v}$ respectively. Velocity gradient = δv/δy.

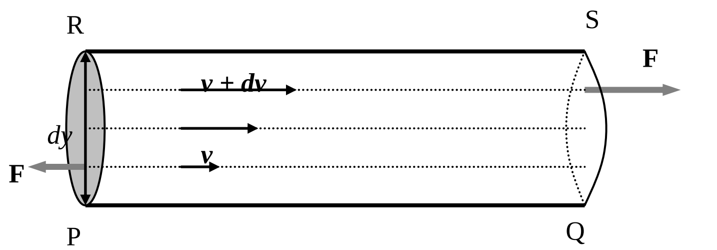


Fig. 5-9

If ***F*** is the tangential force per unit area required to maintain this velocity, then the lower layer exerts an equal and opposite tangential retarding force on the faster upper layer as it experiences an equal and opposite tangential force. This tangential retarding force is proportional to the relative velocity between the two layers, the contact area A between the two layers but inversely proportional to the absolute distance between the layers i.e.,

$$F = \eta \frac{\partial v A}{\delta y} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.7)$$

Thus:

$$\eta = \left(\frac{F}{A}\right)\frac{\delta y}{\partial v} = \frac{\text{tangential stress}}{\text{velocity gradient}} = \left(\frac{F}{A}\right)\Big/\left(\frac{\partial V}{\partial y}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5.8)$$

where η is the coefficient of viscosity (dependent on the nature of fluid).

Viscosity (η) can also be defined as the tangential force per unit area of fluid which resists the motion of one layer over another when the velocity gradient is unity. Unit of viscosity is Decapoise (= 10 poise). η_{H_2O} = 1.1 x 10^{-3} N s m^{-2}; η_{blood} = 2.1 x 10^{-3} N s m^{-2}.

5.3.1 Variation of Viscosity with Temperature

Viscosity decreases with increase in temperature e.g., η_{H_2O} = 1.01 x 10^{-2} Poise at 20^oC and 4.7 x 10^{-3} Poise at 60^oC. During cold weather (winter), the motor oil in automobile engines becomes very much viscous, and the large friction then makes it difficult to start the engine. No definite relation exists between viscosity and temperature but various empirical formula have been suggested of the form

$$Log\,\eta = a + \frac{b}{T} \qquad \text{.....................................(5.9)}$$

where a and b are constants and T is the absolute temperature.

For gases, viscosity increases with temperature (see Kinetic theory of gases) according to

$$\eta = a\eta_o T^{\frac{1}{2}} \qquad \text{.....................................(5.10)}$$

where η_o is the viscosity at 0^oC.

5.3.2 Strokes Law

If a small spherical object is allowed to fall through a viscous medium, then, *the viscous retarding force (F) on the object will be directly proportional to the coefficient of viscosity η of the fluid, the velocity v of the moving object its radius r and the density ρ of the medium*. Combining all these factors we have

$$F = Kvr^a \eta^b \rho^c$$

where *K* is a constant and *a, b* and *c* are dimensional coefficients.

By using dimensional analysis (i.e., putting dimensions of the quantities on both sides of the above equation, it can be shown that

$$M^1L^1T^{-2} = \left(L^1T^{-1}\right)\left(L^a\right)\left(M^bL^{-b}T^{-b}\right)\left(M^cL^{-3c}\right)$$

Solving gives *b = 1, c = 0, a = 1.* Thus

$$F = 6\pi\eta vr \qquad \text{[Stokes law]} \qquad \text{.....................................(5.12)}$$

where K = 6π

5.3.4 Applications of Strokes Law

Stokes law is widely used to determine densities of unknown fluids

In principle, if a small object (e.g. steel ball of radius r, density σ and mass M) is released in a viscous liquid of density ρ and allowed to fall vertically under gravity (see Fig. 5.10), the object accelerates at first, but its velocity soon reaches a steady value known as *the terminal velocity, V_t*.

At this point the viscous force, ***F*** (which increase with the velocity of the body) and the resulting upthrust, ***U***, together equal to its weight, ***Mg*** i.e.,

$$Mg - (F + U) = 0 \qquad \text{..............................(5.13)}$$

But From Strokes law ***F*** = $6\pi\eta r v_t$, and

U due to liquid = wt. of fluid displaced by the object
= Volume of ball × fluid density (ρ) × g

$$= \frac{4\pi}{3} r^3 \rho g$$

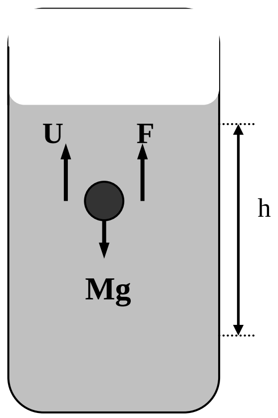
Wt (***W***) of ball = Mg = volume of ball × density of ball (σ) × g

$$= \frac{4\pi}{3} r^3 \sigma g$$

$$\Rightarrow \quad \frac{4\pi}{3} r^3 \sigma g - \frac{4\pi}{3} r^3 \rho g - 6\pi\eta r v_t = 0$$

Solving for ρ gives

$$\rho = \sigma - \left(\frac{\eta v_t}{2r^2 g} \right) \qquad \text{............................(5.14)}$$



From Newton's 2nd law; F = Ma

If Mg > (U + F) ⇒ ball falls faster

i.e. Mg – U – F = Ma

If Mg = (U + F) ⇒ a = 0
⇒ vel constant

Fig. 5.10

Hence the density of an unknown fluid can be found by first determining the terminal velocity, V_t. The terminal velocity is determined by timing the time taken for the ball to fall a given distance *h* (Fig. 5-10).

Strokes formula can also be used to find the rate of fall of an ion in an electric field. The change on the ion is calculated as in Millikan's experiment.

5.3.4 Poiseuille's Equation

If a perfect fluid, with negligible viscosity is initially flowing in a long horizontal pipe, the fluid will continue to flow forever, at constant velocity, in accordance with Newton's 1st law. However, for a real (viscous) fluid flowing in a similar pipe, it will gradually slow down and stop due to frictional resistance generated by viscous forces. To keep a real fluid flowing, a pressure difference between the ends of the pipe must be provided so that the excess pressure at the upstream pushes the fluid along and overcomes the frictional resistance.

According to *Poiseuille's equation*, the pressure difference (P_1 -P_2) required to maintain the flow along a tube is proportional to the viscosity (η), the length of pipe (l) and the rate of flow or delivery ($d\mathbf{v}/dt$), and inversely proportional to the fourth power of the radius (r) of the tube,

$$P_1 - P_2 = \frac{8l\eta}{\pi r^4}\frac{\Delta \mathrm{V}}{\Delta \mathrm{t}} \qquad \text{……………………….…………...….(5.15)}$$

where ΔV/Δt is the volume delivered per second.

The dependence on r^4 implies that for the same rate of delivery, a much larger pressure difference is required to push a viscous fluid through a small pipe than through a large one. For example, a decrease of radius by a mere 10% (or by a factor of 0.9) requires an increase of the pressure difference by a factor of $\frac{1}{(0.9)^4} = 1.5$ if a given rate of delivery is to be maintained. This explains why the blood flow in the human circulatory system is so sensitive to slight reductions of the diameter of the arteries. A slight buildup of plaque in the arteries implies that the heart has to work harder, to provide this higher pressure.

5.3.6 Viscosity and Blood Flow

In the human circulatory system, the work done by the heart to pump blood of volume dV into the aorta and the circulatory system at pressure $P_a = P_a dV$ where P_a is the arterial pressure.

The rate at which the heart does work against viscous friction in the circulatory system i.e. power of the heart is

$$P = \frac{dW}{dt} = P_a \frac{dV}{dt} \qquad \text{……………………………(5.16)}$$

Thus a reduction in the diameter of arteries increases P_a and from Eqn. (5.15), this implies the heart must deliver more power. This places a mechanical load on the heart and will require an increase in oxygen and in chemical energy consumed. The result may be heart failure (Cardiac arrest)

High blood pressure may occur as a result of

-Physical stress e.g., heavy mechanical exercise or emotional stress

-Accumulation of plaque in the arteries (atherosclerosis). This is a case of chronic high blood pressure (hypertension), which is common in old age. It decreases the inner radius of the arteries but increases the viscous resistance.

5.3.7 Critical Velocity and Reynolds Number

The critical velocity of a liquid is that velocity of flow above which the flow ceases to be streamline. Its value depends on

(i) The coefficient of viscosity, η, of the fluid

(ii) The density (ρ) of the fluid

(iii) The radius (r) of the tube. Thus

$$V_c = K\eta^a \rho^b r^c$$

where *K* is a constant and *a, b* and *c* are dimensional coefficients.

Putting dimensions of various quantities on both sides of the equation we have

$$LT^{-1} = \left(ML^{-1}T^{-1}\right)^a \left(ML^{-3}\right)^b (L)^c$$

From the principle of homogeneity of dimensions *a = 1, b = -1, c = -1.* Thus

$$V_c = K\frac{\eta}{\rho r} \qquad \text{…………………………………………..(5.17)}$$

where *K* is the Reynolds number (*K* = *Re* = 2000).

It follows from Eqn. (5.17) that narrower tubes, low density and high fluid viscosity, helps in producing orderly motion.

Re is a dimensionless quantity and its value determines the type of flow e.g. in the case of a cylindrical pipe,

If $Re < 2200 \Rightarrow$ steady (laminar) flow
If $Re \cong 2200$ ($\boldsymbol{v_c}$) $\Rightarrow$ flow is unstable
If $Re > 2200 \Rightarrow$ flow is turbulent.

Worked Example

1. Two drops of water of the same size are falling vertically through air with terminal velocity of $1ms^{-1}$. If the two drops combine to form a single drop, cal the terminal velocity.

Solution

Let r = radius of each drop and r_1 that of the combined drop, then

$$\frac{4\pi}{3}(r_1)^2 = 2(\frac{4\pi}{3})r^3 \quad \Rightarrow \quad r_1 = 2^{1/3}\,r$$

If v_t is the terminal velocity of each drop and v_1 that of the combined drop then according to stroke's law

$$v_t = \frac{2r^2(\rho - \sigma)}{9\eta}g \quad \text{and} \quad v_1 = \frac{2r_1^2(\rho - \sigma)}{9\eta}g \quad \Rightarrow \quad v_1 = 1.588\ ms^{-1}.$$

Questions for Discussion

1. (a) According to popular belief, blood is denser than water. Is this true?
   (b) Explain what holds a suction cup on a smooth surface.

2. How does a balloonist control the ascent and descent of a hot-air balloon?

3. Suba divers have survived short intervals of free swimming at depths of 430m. Why does the pressure of the water at this depth not crush them?

4. Why do some men or women float better than others do? Why do they all float better in salt water than in plain water.

5. Explain the following
   (a) would you float if your lungs were full of water
   (b) how does a submarine dive? Ascend?
   (c) when you release a bubble of air while underwater, the bubble grows in size as it ascends. Explain.

6. Face masks for scuba divers have two indentations at the bottom into which the diver can stick the thumb and forefinger to pitch his nose shut, That is the purpose of this arrangement?

7. Use the Bernoulli's principle (include a labeled flow pattern sketch) to explain
   (a) why two nearby ships proceeding side by side might be in danger of colliding
   (b) why accumulation of ice on airplane wings might be dangerous.

8. Why are the continuity and Bernouilli's equations only approximately valid for the flow of air?

9. A girl standing in a subway car holds a helium balloon on a string. Which way will the balloon move when the car accelerates?

TUTORIAL 5.1

1. A hypodermic syringe has a needle of length 4cm and internal radius 0.2 mm. The syringe contains 4 cm^3 of distilled water. What pressure must one apply to the water to squirt it out of the syringe in 10s.

2. Commercial jetliners have pressurized cabins enabling them to carry passengers at a cruising altitude of 10,000m. The air pressure at this altitude is 210 mm-Hg. If the air pressure inside the jetliner is 760 mm-Hg, what is the net outward force on a 1m × 2m door in the wall of the cabin?

3. A 'suction' pump consists of a piston in a cylinder with a long pipe leading down into a well. What is the maximum height to which a pump can 'suck' water?

4. You can walk on water if you wear very large shoes shaped like boats. Calculate the length of the shoes that will support you; assume that each shoe is 0.3m × 0.3m in cross-section.

5. A pipe is running full of water. At a certain point A it tapers from 0.6m diameter to 0.6m diameter at B. The pressure difference between A and B is 1m of water column. Find the rate of flow of water through the pipe.

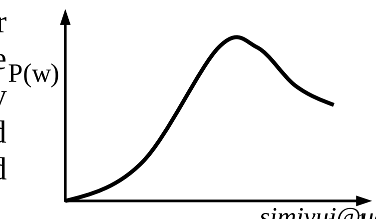
6. A lead ball is placed on the surface of oil and released. Explain why the ball accelerates at first and soon reaches a steady velocity. Briefly explain how you would measure the steady velocity

7. A lead ball is placed on the surface of a viscous oil and released.
   (i) State the forces acting on the ball as it falls through the oil
   (ii) Which of these forces varies during the fall and explain why it varies
   (iii) Sketch a graph showing the variation of the velocity of the ball with time from the moment it was released. Also draw a second graph showing this variation if the oil was replaced with water.

Questions for Discussion

1. Discuss the operation of a centrifuge from the view-point of
   (a) an initial frame of reference, and
   (b) a rotating frame of reference.
2. When a mammal is running on a level ground, a large amount of energy is used in accelerating the limbs. By considering moments of inertia, discuss the likely distribution of muscle in fast mammals.

3. The diagram shows how the power of a motor car depends upon the angular velocity of the crankshaft. Use the graph to explain why maximum torque occurs at a lower engine speed than maximum power. Discus the ranges of road



speed for which a motorists can obtain greater acceleration by engaging a lower gear.

4. (a) What is the effect of a helicopters' fuse-lage on the rotation of the main rotor? How can a tail rotor counteract this effect.
   (b) Explain how it is possible to distinguish between a raw egg and a hard-boiled one by spinning each on a table.

5. In the earth-moon system, what effect do tidal forces have on the spin angular momentum of the earth? What is the consequence of this.

6. (a) If a cat is dropped upside down and with no rotation, how can it turn over so as to land on its feet?
   (b) How can a skater and a diver increase their angular speeds?

7. Explain the following observations
   (a) A buttered piece of bread always lands on its buttered side.
   (b) Rifle bullets are given a spin about their axis by spiral grooves (`rifling') in the barrel of the gun. What is the advantage of this?
   (c) A tight ropewalker uses a balancing pole to keep steady. How does this help.

Suggested Further Reading

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25. OHANIAN H. C., (1994). **Principles of Physics**, *I^st Ed*., W.W. Norton & Company Inc, New York, London, pp
26. NELKON M. and PARKER P., **Advanced Level Physics**, *7^th Ed*.
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APPENDIX

Selected Physical constants

Boltzmann constant,	$\boldsymbol{k} = 1.38\times10^{-23}$ J K^{-1}.
Acceleration due to gravity,	$\boldsymbol{g} = 9.81$ ms^{-2}.
Atomic mass unit,	$1\mu = 1.66 \times10^{-27}$ Kg.
Avogadro's number,	$\boldsymbol{N_A} = 6.02 \times10^{-23}$ mol^{-1}.
Universal gas constant,	$\boldsymbol{R} = 8.31$ J K^{-1}.
Atmospheric pressure,	***1 atm*** $= 1.01 \times10^{5}$ NM^{-2}.
Density of water,	$\rho = 1000$ Kg m^{-3}.
Heat of vapourization of water,	$\boldsymbol{Q} = 539$ Kcal Kg^{-1}.
Heat of fusion of ice,	$\boldsymbol{Q} = 79.7$ Kcal Kg^{-1}.
Mechanical equivalent of heat,	1 Cal = 4.19 J.
Specific heat capacity of water,	$\boldsymbol{c} = 4.19$ KJ Kg^{-1} K^{-1}.

Unit Prefixes

Fraction	*Prefix*	*Symbol*	*Multiple*	*Prefix*	*Symbol*
10^{-3}	milli	m	10^{3}	kilo	K
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	Femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E